Terms of Trade and Fiscal Sustainability when the Sovereign Exploits a Natural Resource

Leandro Andrian
Marcelo Oviedo

December 2013

Inter-American Development Bank
Country Department Andean Group
Terms of Trade and Fiscal Sustainability when the Sovereign Exploits a Natural Resource

Leandro Andrian
Marcelo Oviedo

Inter-American Development Bank
2013
The opinions expressed in this publication are those of the authors and do not necessarily reflect the views of the Inter-American Development Bank, its Board of Directors, or the countries they represent.

The unauthorized commercial use of Bank documents is prohibited and may be punishable under the Bank's policies and/or applicable laws.

Copyright © 2013 Inter-American Development Bank. This working paper may be reproduced for any non-commercial purpose. It may also be reproduced in any academic journal indexed by the American Economic Association’s EconLit, with previous consent by the Inter-American Development Bank (IDB), provided that the IDB is credited and that the author(s) receive no income from the publication.

JEL Classification: E32, E37, E62, F34, H63
1. Introduction

When it comes to ponder fiscal policy in Andean countries there are no reasons to think about dark skies that could threaten fiscal sustainability in the short to medium run, even under moderate stress tests. Increasing commodity prices, the rise of emerging countries as prosperous trade partners that offset the falling external demand from growth-stagnant industrial countries, low interest rates, and capital inflows to the region shape a sky without significant clouds in the near future. With slight internal variations, the fiscal strength of Andean countries looks even better if one factor in that sovereigns in these countries own and exploit the sources of the natural resources that the new, prosperous, emerging world is demanding.

As shown in Table 1, the ratios of public debt to GDP have fallen between 10 and 20 percentage points between 2003 and 2011. Strong growth in the region was central to that: compared to the 1990's, real GDP growth is on average 1.6 percentage points higher during the 2000's. Furthermore, the high relative value of Latin American exports to the rest of the world added another strong pillar to fiscal substantiality in the region. Figure 1 shows the terms of trade of the LAC7 countries for the period 1990-2011 and makes it evident the striking contrast between the first and second decade of that period. Two related reasons justify qualifying the improvement of the terms of trade as a strong pillar of fiscal sustainability. One is that fiscal revenues coming directly from natural resources account for considerable fractions of total fiscal revenues: 44% in Bolivia, 11.5% in Colombia, 41.4% in Ecuador and 14.4% in Peru. And the second one is the high correlation between the terms of trade of these countries and their respective fiscal revenues, ranging from 0.79 in Ecuador to 0.90 in Peru.

Despite the actual environment, sound fiscal policy requires considering more than moderate stress tests and assessing what would happen if the external environment becomes less

---

1 We appreciate the comments of Leopoldo Avellán, Arturo Galindo and an anonymous referee.
2 Izquierdo and Talvi (2011) group Latin American countries in two clusters, one called the Brazilian cluster to which the Andean countries belong and which is characterized by having stronger growth fundamentals than those in the Mexican cluster.
3 LAC7 countries TOTs are shown as a representative average of TOTs in Latin America. The countries included in LAC7 are the largest seven in the region, namely Argentina, Brazil, Chile, Colombia, Mexico, Peru, and Venezuela.
favorable to the region than what the present brings in. A strong terms-of-trade reversal should be among the scenarios to be factored in by a policymaker aware of that terms of trade were 4.9 times more volatile in Latin America than in industrial countries during the 1990-2011 period and that the developed world, specially Europe, still faces serious financial risks and is under considerable chances of suffering a deeper macroeconomic downfall.\(^4\) It is then valid to ask whether current fiscal policies would be sustainable in an environment less propitious than the present. What would then happen to public debt? Would it grow enough so that issuing new debt would become more and more difficult? or would it remain under control? What policymakers should be able to do in that scenario? Are their responses viable from the economic viewpoint?

Answering these questions requires incorporating all exogenous and endogenous effects of a macroeconomic shock into the fiscal analysis. For instance, consider an adverse terms-of-trade shock that slashes the price of a natural resource largely exploited by the government. There will be a direct impact on government finances because the value of what the government produces and from where it obtains a large share of its revenues falls. But complementing this, there are several indirect effects. First, production decisions regarding the exploitation of the natural resource following the shock should be incorporated as they will feedback into the fiscal balance. And so has to be done with the private sector participation in the exploitation of the natural resource. On another margin, the shock will reduce output, consumption and investment. As the sovereign is taxing consumption and income, there will be another indirect effect of the shock on government finances as tax bases shrink. Aggregating all direct and indirect effects in macroeconomics calls for a general equilibrium model. The goal of this paper is to provide such a tool, by building a stochastic model that incorporates all sources of fiscal revenues in Peru and Colombia.

The next section describes some stylized facts for the 2000-2011 period in Andean countries regarding the evolution of ToT, public sector and external accounts. Then, after reporting business cycle statistics to first validate the model, impulse response functions are computed to understand the general equilibrium responses to a term of trade shock and its effect on the fiscal balance and the stock of public debt in Peru. The paper then shows possible fiscal policies that

\(^4\) The higher terms-of-trade volatility refers to the fact that, as a percentage of the average, the standard deviation of the terms of trade is 4.89 larger in Latin American and Caribbean countries than in the G7 countries.
would guarantee the sustainability of public debt if a strong reversal in the terms of trade occurs. In particular reactions functions for the tax rates and government expenditures that avoid public debt to grow to unsustainable levels are considered for Peru and Colombia. The paper closes with another validation exercise in which the fiscal policy implemented in Peru when the country faced a severe, adverse terms-of-trade shock in 2009, is included in the model to obtain results highly similar to what has been observed in this country following the shock. With some caveats, the exercise is repeated for Colombia to also obtain encouraging results about the use of a general equilibrium model to study the fiscal consequences of macroeconomic shocks and the options open to fiscal policymakers to withstand the effect of the shock.

2. Terms of Trade, Public Sector and Exports in Andean Countries

This section describes some stylized facts for the period 2000-2011, and provides some economic intuition for the issues analyzed in the following sections. We analyze four Andean countries: Bolivia, Colombia, Ecuador and Peru. For this set of countries the evolution of ToTs and fiscal accounts are quite remarkable. For example, depicts the evolution of both the average ToTs and the average ratio of fiscal revenue to GDP, both series normalized as indices (2000 = 100). This figure suggests a how close is the co-movement of both series; in fact the simple correlation between ToTs and revenue/GDP is 0.96.

A totally different picture arises when the evolution of ToTs is compared to the ratio of public debt to GDP (see Figure 3). In this case, an opposite trend appears. That is, while the ToTs series evolves according to an ascendant trajectory, the debt to GDP ratio displays a descendent path. The correlation between both series is -0.89.

An even more compiling example of these trends can be seen during 2009. At this time the global financial crisis hits its maximum, the ToTs for Andean countries fell 11.3%; fiscal revenue to GDP fell also by 7.6%, while public debt to GDP increased by 8.3%. The dependence of Andean governments to income coming from commodities becomes clear. Related to this, Figure 4 suggests the difference between the overall balance and the non-commodity balance in all countries is significant, averaging is 8.6 percentage points of GDP.
Much in the same vein, the external sector remains highly dependent on export's commodity. For the Andean countries analyzed, more than 40% of total exports depend on a single commodity, which is also the same source of commodity related fiscal revenue. The ratio of export's commodity to total export ranges from 44% in Bolivia and Colombia to 54% in Ecuador and 60% in Peru. This leads to a noticeable co-movement between the evolution of the international price of the main commodity exported and ToTs for Andean countries (see Figure 5). The correlation between these two series is 0.98, showing the importance of the international price of commodities in the evolution of ToTs\(^5\).

### 3. A Small Open Economy with a Natural Resource

Consider an artificial small open economy that connects to the rest of the world through output and financial markets and that is populated by three types of agents: households, firms, and the government.

Each atomistic household supplies labor services and owns the capital stocks used in two production sectors. That household is representative of an infinite number of families that are identical to each other and take consumption, investment, and savings decisions, as well as the decisions regarding the sectoral allocation of labor and capital. The choices are aimed at maximizing welfare or utility over a long horizon that is represented as an infinite horizon. Furthermore, the household is the financial link between the economy and the rest of the world for it represents the sector that borrows and lends in international financial markets. International financial trading is limited to one-period, non-contingent bonds issued in units of the exportable good.

Moving to the production side of the economy, the model assumes the existence of an infinitely large number of firms that operate in a perfect competitive environment and are grouped in two sets. One produces nontraded goods and the other exportable goods. Importable goods are not produced domestically. Firms are profit-maximizing agents that take input and output prices as market determined, and period by period decide on optimal input utilizations.

\(^5\) A regression between ToTs and the international price of commodity shows a \(R^2\) equal to 0.97
Whereas capital goods in the nontradable sector are produced by allocating nontradable goods to their production, in the exportable sector, capital goods are linearly produced with imported inputs.

The government is the third type of agent in the economy. It obtains public revenues by taxing consumption and income and by exploiting a natural resource. Although throughout the paper this natural resource is referred as oil, the reader should keep in mind that oil stands for the natural resource relevant for each of the Andean countries. Whereas it actually is oil in Colombia, Venezuela and Ecuador, it is mining in Peru and Bolivia. Tax rates are initially fixed but a variant of the model with state-contingent tax rates that guarantee fiscal sustainability is analyzed below. The price of the natural resource is determined in the rest of the world and production decisions respond to an exogenous supply function whereas an oil production function determines how much of the importable good must be used for oil production.

The government uses its resources to provide a flow of services which are a component of the demand for nontraded output and which represent the provision of public goods in practice. Additionally, the government extends transfers to the households and the size of the transfers depend on the price of oil in order to capture the effect of changes in the price of the country's key natural resource on the private sector income.

At business-cycle frequency, changes in oil prices and fluctuations in aggregate consumption and income impact on government finances. Faced with fixed amounts of public goods to be provided to the household, fluctuations in fiscal revenues force the government to resort to public debt to balance the budget.

The bonds issued by the government are domestic and only bought by the households, although a non-arbitrage condition guarantees that both government and international bonds hold the same rate of return. Notice that households are only interested in their net asset position, disregarding of the composition between international and government bonds. When the government issues bonds, they are acquired by the households. Without further action, the transaction would increase the net asset position of the latter. If the households want to keep their
previous net asset position, they will neutralize the debt issued by the government by issuing more of their own debt in international bonds markets.\(^6\)

Before entering into more technical details, it is worth to summarize the model's components as well as the role of oil in the artificial economy. There are four goods: i) an exportable good unrelated to the country's natural resource; ii) an importable good; iii) a nontrade good; and iv) oil, whose production is also exportable. There are three types of agents: households, firms, and the government. Domestic markets determine the price of nontraded goods, public bonds, capital and labor; for the countries under study, it is sensible to assume that the price of traded goods as well as the world, constant, interest rate are exogenously determined abroad. Terms-of-trade shocks are captured by disturbances to the relative price of importable goods and by disturbances to the relative price of oil; both prices are expressed as ratios to the price of the exportable good which is the *numeraire*.

Although oil is monopolistically exploited by the government it also adds to the household's income through government's lump-sum, oil-related transfers. As described with further details below, an oil-price hike will increase both government's revenues and transfers, and the latter in turns increases the household income.

### 3.1 Households

The representative atomistic household chooses sequences of the capital stocks to be used in exportable-good and nontraded-good productions; international and public bonds; consumption of each type of good; and leisure so as to maximize the lifetime expected utility \(U_t\), where:

\[
U_t = \sum_{s=0}^{\infty} E_t \theta_t [u(c_{t+s}, l_{t+s})]
\]

\(^1\)

---

\(^6\) Thus, the assumption that only the household is allowed to borrow and lend abroad is made without loss of generality. In other words, it can be shown that the optimal allocations obtained under the present arrangement are the same as the allocations that would optimally arise in another arrangement where both the government and the household issue bonds in international financial markets. To see this, define \(b^h = b' + b^e\) as the household net asset position. If the government wants to issue more bonds but the household is not willing to modify its net asset position, \(b'\) should fall by the same amount that \(b^e\) increases, which is tantamount to saying that the government is directly borrowing abroad.
where \( E_t \) is the expectation operator that conditions on the information available at time \( t \) and \( u \) is an instantaneous utility function that is strictly concave and increasingly differentiable. Its arguments are the consumption of goods, \( c_t \), and leisure, \( l_t \). The former is a linear-homogeneous function \( \bar{c} \) of the consumption of exportable, importable, and nontraded goods, respectively, \( c_t^x \), \( c_t^m \), and \( c_t^n \), namely \( c_t = \bar{c}(c_t^x, c_t^m, c_t^n) \), where the exportable good excludes the natural resource good. The discount factor \( \theta_t \) is defined by the function \( \theta_t = \Theta([c_s]^s_{s=0}, [l_s]^s_{s=0}) \) that depends positively on each component of the whole history of allocations of consumption and leisure. The reason a function is used instead of a typical exponential discount factor is a technicality of small-open-economy models that is explained in detail in Schmitt-Grohé and Uribe (2003).

The household faces a time constraint. The sum of leisure time and the time allocated to produce exportable and nontraded goods, respectively \( \ell_t^x \) and \( \ell_t^n \), cannot exceed the time endowment \( T \):

\[
l_t + \ell_t^x + \ell_t^n \leq T; \quad t \geq 0
\]

A flow budget constraint further limits the choices of the household by equating sources to uses of resources. The sources leading to the household's after-tax, or net income are:

- **Labor income**: \( W_t^x \ell_t^x + W_t^n \ell_t^n \)

- **Capital income**:
  \[
  (R_t^{ki} / p_t^m - 1 - \delta^i)k_t^i p_t^m + (R_t^{kn} / p_t^n - 1 - \delta^n)k_t^n p_t^n
  \]

- **Financial income**:
  \[
  (R - 1)(b_t + b_t^s)
  \]

- **Net income**:
  \[
  (1 - \tau_t)(\text{Labor inc.} + \text{Capital inc.} + \text{Financial inc.}) + T_t^e + \bar{T} + \Omega
  \]

where \( k_t^i \) and \( \ell_t^i \), \( i = x, n \), is the stock of capital and labor employed in industry \( i \) at time \( t \), which receive the sectoral wage rates and the gross returns on capital, \( W_t^i \) and \( R_t^{ki} \) respectively. Note that capital income is computed using the net rate of returns on capital after accounting for
the rate of depreciation of the capital stocks, \( \delta_i, i = x, n \), and the market prices of capital goods \( p_i^m \) and \( p_i^n \). Following Mendoza (1995), we assume that the stock of capital goods used in the non-traded sector is produced in that sector at the price \( p_i^n \), while the stock of capital used in the exportable industry is imported at the cost (price) \( p_i^m \). The household's stock of international and government bonds are \( b_i \) and \( b_i^g \), respectively. The fixed gross world interest rate is \( R \). Labor and capital income are taxed at the rate \( \tau_i \). In addition to the income sources in (3a) to (3c), the after-tax income in includes public transfers from oil \( T_i^e > 0 \), whose meaning is explained below; other fixed, government net transfers \( \bar{T} \) used for calibration purposes\(^7\); and foreign remittances \( \Omega \). Including remittances is necessary for an accurate calibration of the model to Andean countries because this component of national income has recently been as large as 5% of GDP.\(^8\) For simplicity, remittances in the model are a time-and-state invariant quantity of exportable goods received from the rest of the world.

The household's uses of resources are:

Consumption of goods:
\[
(1 + \tau^e)(p_i^m c_i^m + c_i^x + p_i^n c_i^n)
\]

(4a)

Gross acquisition of new capital goods:
\[
p_i^m \left[ k_{i+1}^x - k_i^x + AC_i^x \right] + p_i^n \left[ k_{i+1}^n - k_i^n + AC_i^n \right]
\]

(4b)

Net acquisition of financial assets:
\[
b_{i+1} + b_i - \bar{T}_i^\tau - b_i^g
\]

(4c)

As the exportable good is the numeraire and it is normalized to one, \( p_i^m \) and \( p_i^n \) are the relative prices of importable and nontraded goods with respect the price of exportable goods. Likewise, \( p_i^e \) is the relative price of oil which is determined in world oil markets. The government taxes consumption at the rate \( \tau^e \). Expenditures on capital goods comprise the net acquisition of capital goods in both the exportable-goods and nontraded-goods industry as well as the capital adjustment costs \( AC_i^i, i = x, n \). Including adjustment costs responds to the necessity

---

\(^7\) \( \bar{T} \) could be negative which would represent a lump-sum tax

\(^8\) According to country statistics, for the period 1990-2011, remittances are in the range of 2% to 5% of GDP.
of distinguishing between financial and physical capital and of avoiding a variability of investment that otherwise would largely exceed what is observed in practice.

**Functional Forms and Optimality Conditions**

In the objective function (1), \( u \) is a standard constant-relative-risk-aversion utility function \( u(x_t) = (1-\sigma)^{-1} x_t^{-\sigma}, \) where \( \sigma > 1 \) is the coefficient of relative risk aversion and \( x_t \) is the composite good involving \( c_t \) and \( l_t \):

\[
x_t = c_t - \frac{\omega_0}{\omega_1} (T - l_t)^{\omega_1}
\]

where the labor supply \( (T - l_t = \ell^n_t + \ell^x_t) \) enters as a power function.\(^9\) The parameter \( \omega_1 > 0 \) determines the elasticity of labor supply; which is equal to \((1-\omega_1)^{-1}\). On the other hand, the composite good \( c_t \) is defined as a constant-elasticity-of-substitution (CES) function nesting a Cobb-Douglas function:

\[
c = \tilde{c}(c^m, c^n, c^x) = \left[ \chi \left( (c^m)^a (c^n)^{1-a} \right)^\eta + (1-\chi) (c^n)^{-\eta} \right]^{\frac{1}{1+\eta}}
\]

Here, \( \eta > -1 \) determines the elasticity of substitution between traded and nontraded goods, \((1+\eta)^{-1}\); \( a \in [0,1] \) and \((1-a)\) are the shares of exportables and importables in the total consumption of traded goods; and \( \chi \in [0,1] \) is a weight parameter reflecting the importance of traded and nontraded goods in total utility. Although tedious to derive, it is known that composite goods \( c_t \) and \( c_t^T = (c_t^x)^a (c_t^m)^{1-a} \) each has an associated consumption-based price index, denoted here as \( p_t^C \) and \( p_t^T \), such that:

\[
p_t^T c_t^T = p_t^m c_t^m + c_t^x \quad \text{and} \quad p_t^C c_t = (p_t^m c_t^m + p_t^n c_t^n + c_t^x)
\]

---

\(^9\)In the definition of the composite good \( c_t \), note that its dependency on leisure comes from the time constraint in (2), from where it can be seen that \( \ell^n_t + \ell^x_t = T - l_t \).
These indexes are:

\[ p_i^T = a^{-a} (1-a)^{-1} (p_i^m)^{1-a} \quad \text{and} \quad p_i^C = \left[ \frac{1}{1-a} \left( \frac{c_i^1}{c_i^m} \right)^{1-a} + (1-\chi) \left( \frac{c_i^n}{c_i^m} \right)^{1-a} \right]^{\frac{1}{1-a}} \]  (7)

The consumption-based price index \( p_i^C \) can be interpreted as a consumer price index although all its changes are driven by real forces and not by monetary impulses. The index can be also understood as the real exchange rate.\(^{11}\)

The function form of the adjustment cost function is \( AC_i^i = \frac{\phi}{2} (k_i^i - k_i^s)^2 \), \( i = x, n \). By optimizing on the uses of its resources, the household equates the marginal rate of substitutions (MRS) between any two goods to the ratio of their corresponding prices:

\[ \text{MRS}_{x,x} = p^m \Rightarrow \left( \frac{1-a}{a} \right) \left( \frac{c_i^x}{c_i^m} \right) = p_i^m \]  (8a)

\[ \text{MRS}_{x,x} = p^a \Rightarrow \left( \frac{1}{a} \right) \left( \frac{1-\chi}{\chi} \right) \left[ \frac{c_i^x}{c_i^m} \right]^{\frac{1-a}{1-a}} \left( \frac{c_i^x}{c_i^m} \right)^{\frac{1-a}{1-a}} = p_i^a \]  (8b)

An oil-price hike increases the household's net income because of the government oil dependent transfers \( T_i^e \). As \( p_i^m \) is exogenously determined, eq. (8a) shows that, \( \text{ceteris paribus} \), the consumption of exportable and importable must experience the same proportional increase. How the demand for nontraded goods adjusts depends, among other things, on the elasticity of substitution between traded and nontraded goods and on the share of traded and nontraded consumption in total consumption. By totally differentiating (8b) under a constant \( p_i^a \), one can see that the consumption of traded goods falls proportionately more (less) than the consumption of nontraded goods if the ratio \( c_i^T / c_i^s \) is higher (lower) than \( 1 + \eta \).

The price \( p_i^a \) is, however, endogenous and so it does not remain unchanged when the economy is hit by an oil-price shock. Although \( p_i^a \) is expected to fall as the consumption

\(^{10}\) The steps to derive the consumption-based-price indexes can be seen, for example, in Obstfeld and Rogoff (1997).

\(^{11}\) For details on this equivalence, see Mendoza (1995).
demand for nontraded goods falls, its final response also depends on the other components of the demand for nontraded goods as well as on the supply response of the firms.

A rise in $p^m_t$ is interpreted as a worsening of the terms of trade in the artificial economy. When $p^m_t$ rises, the household rebalances its consumption basket substituting importable goods with exportable and nontraded goods. The price of nontraded goods increases and so does $p^c_t$.

Labor-leisure decisions arise from the equalization of the marginal rate of substitution of consumption for leisure to the after-tax wage rate in the nontraded industry, adjusted to incorporate the cost of buying non-oil consumption goods:

$$\omega_0 = (\ell^n_t + \ell^n_t)^{\alpha_1} = W_t^i \left[ \frac{1 - \tau^l}{1 + \tau^c} \right] \frac{1}{p^c_t}$$

(9)

The endogenous discount factor can be written recursively as:

$$\theta_0 = 1, \theta_{t+1} = \beta_t \theta_t$$

(10)

where $\beta_t = \exp \{ \nu \ln[1 + c_{t-1} - \omega_t / \omega_t(\ell^n_{t-1} + \ell^n_{t-1})^a] \}$. As for the investment and saving decisions, the optimality conditions for bonds (international and government) and capital in the exportable and importable-goods industries, all equate the marginal utility cost to the marginal utility benefit of saving in each of the four assets:

$$\lambda_t = \beta_t [1 + r(1 - \tau^l)] E_t \lambda_{t+1},$$

(11)

$$\lambda_t p^n_t [1 + \phi^n (k^n_{t+1} - k^n_{t})] = \beta_t E_t p^n_{t+1} \lambda_{t+1} (1 - \tau^l) (\frac{R^n_t}{p^n_t} - \delta^n + 1 + \phi^n (k^n_{t+2} - k^n_{t+1})),$$

(12)

$$\lambda_t p^m_t [1 + \phi^m (k^m_{t+1} - k^m_{t})] = \beta_t E_t p^m_{t+1} \lambda_{t+1} (1 - \tau^l) (\frac{R^m_t}{p^m_t} - \delta^m + 1 + \phi^m (k^m_{t+2} - k^m_{t+1})),$$

(13)

where $\lambda$ is the Langrange multiplier of the household's resource constraint interpreted as the marginal utility of consumption for exportable goods. It is worth to notice that the two capital Euler equations are expressed in terms of the their reposition cost ($p^n$ and $p^m$), but since the
capital's rental price \( (R^i, i = x, n) \) is expressed in terms of the exportable good, that requires dividing by \( p^n \) (for non-traded goods) and \( p^m \) (for exportable goods).

### 3.2 The Production Side of the Economy

Firms produce either exportable or nontraded goods and their time \( t \geq 0 \) profit-maximization problem is as follows:

\[
\max_{l^t_x, k^t_x} \left\{ p^t_x A^t_x \left( l^t_x \right)^{1-a_j} - W^t_x l^t_x - (R^t_{k^t_j} - 1) k^t_x \right\}, j = x, n
\]

with the proviso that \( p^t_x = 1 \ \forall t \). Labor and capital, respectively, \( l^t_x, k^t_x \), are the factors of production and \( A^t_x \) is a scaling parameter subject to productivity shocks.

Profits are maximized when firms equate the marginal revenue product of each input to its corresponding input price. The linear homogeneity of the production functions along with the assumption of perfect competition guarantees that in equilibrium profits are equal to zero.\(^{12}\)

### 3.3 The Government

The non-standard introduction of the government in the model responds to the role of natural resources revenues in Andean countries. As explained in Sections 1 and 2, oil (which really means the natural resource indigenous to each country) is a significant source of fiscal revenues and the main export good in each of these countries. The model matches three key facts in that regard: a) the share of oil in total exports; b) the share of oil revenues in total fiscal revenues; and c) the share of oil in GDP.

Starting with the latter, it is assumed that the technology to obtain oil requires using inputs of imported goods and natural resources, with the particularity that no effective input substitution is possible, given rise to \( L \)-shape isoquants. The technology is thus a fixed-coefficients technology, with coefficients \( \zeta^m \) and \( \zeta^e \) with respect to the imported input and natural resources, respectively. Therefore, given an output level of oil, \( y^e, y^e \zeta^m \) of imported goods and \( y^e \zeta^e \) units of natural resources are employed.

\(^{12}\)For this reason, profits were not included in the household's budget constraint.
The external sales of oil, which equal the amount of oil produced, are determined by the following price dependent reaction function:

\[ y^e_t = \varepsilon_1 (p^e_t)^{\varepsilon_2} \]  

(15)

where \( \varepsilon_1 > 0 \) is a constant and \( \varepsilon_2 > 1 \) is the elasticity of oil production and exports with respect to the oil price. Combining this equation with the fixed-coefficients technology, it is straightforward to link the oil price with the amount of imported inputs employed, i.e.

\[ m^e_t = \frac{\varepsilon_1}{\varepsilon_3 m} (p^e_t)^{\varepsilon_4} \]

and to see that the rent accruing to the owner of the natural resource is equal to \( p^e_t y^e_t - p^m_t m^e_t \). A recent study by the IADB (see Casas (2012)) is used here to accommodate the fact that a share of the rent is going to the private sector. The study links the oil price to oil fiscal revenues in Andean countries, \( I^e_t \), through the following reaction function

\[ I^e_t = \varepsilon_3 (p^e_t)^{\varepsilon_4} \]  

(16)

where \( \varepsilon_3 > 0 \) is a constant and \( \varepsilon_4 > 0 \) equals the value of the elasticity of fiscal revenues with respect to \( p^e \) reported in the aforementioned IADB study, and \( I^e_t \) is measured in terms of the exportable good.

To account for the fraction of the oil rent going to the private sector, it is assumed that the government is extending oil related transfers to the private sector, \( T^e_t \), by the following amount:

\[ T^e_t = p^e_t y^e_t - p^m_t m^e_t - I^e_t \]

Thus, \( T^e_t \) is endogenous and driven by both the elasticities of oil exports and of oil fiscal revenues with respect to \( p^e \), and dependent on this price and the production of oil in the country, making that household income jumps whenever there is a positive oil price shock.

The government budget constraint summarizes the government's role in the artificial economy:

\[ p^e_t g^e_t + g^e_t + T^e_t + b^e_t R = \tau^1 [\text{Household Income}] + \tau^e p^e_t c_t + I^e_t + b^e_{t+1} \]  

(17)
where the sources of household income were already detailed. In the above constraint, the uses of resources are the market purchases of nontradable and exportable (non-energy) goods, the gross repayment of the government debt and the oil related transfers to the private sector. The sources of government resources are income and consumption tax revenues, natural resource revenues, and issuance of new debt. An alternative view of the constraint indicates that whenever fiscal revenues fall short of (exceeds) fiscal outlays (including interest payments on public bonds \(((R-1)b^x_i))\), the net stock of government bonds increases (decreases) to cover the deficit (surplus).

An increase in \(p^e_i\) adds to the total fiscal revenue depending on the direct and indirect responses to the oil shock. The direct effect is through the increment of the external sales of oil \(p^e_i (y^e_i)\). The indirect effect depends on how household's income (i.e. the transfers \(T^e_i\)) affects the other household sources of income and its consumption and saving decisions. It is then clear that as the government collects fiscal revenues from different sources, it is necessary to know how the shock affects all tax bases. The complexity of this problem illustrates the benefits of counting with a general-equilibrium model to investigate the public-finance implications of oil-price shocks.

### 3.4 Source of Stochasticity

The model has four sources of uncertainty, two productivity shocks, i.e, \(A_i'\), \(i = n, x\); a non-oil terms-of-trade shock represented by changes in \(p^m_i\); and an oil-price shock represented by changes in \(p^e_i\). The realization of these shocks is summarized by the vector \(X_t = (A^n_i, A^x_i, p^e_i, p^m_i)\).

The stochastic properties of these shocks will be set to capture the sources of price and output fluctuations in two Andean countries. The four sources of uncertainty are assumed to evolve according to a VAR(1):

\[
\ln(X_t) = \Gamma_0 + \Gamma_1 \ln(X_{t-1}) + E_t
\]

(18)

where \(\Gamma_0\) is a 4×1 vector and \(\Gamma_1\) is a 4×4 matrix of autoregressive coefficients, both to be estimated from the data along with the variance-covariance matrix, \(\Sigma\), of the 4×1 vector of innovations \(E_t\).
3.5 Market Clearing Conditions and the Competitive Equilibrium

The model has two input-market clearing conditions, one for capital and another for labor, and two final goods market clearing conditions, one for nontraded and the other for tradable goods, including oil. The latter can be interpreted as an economy-wide resource constraint. In the market for non-traded goods, the equilibrium is reached when output is equal to the sum of public and private expenditures on nontraded goods, including the use of these goods in the production of capital goods, namely:

\[ y^n_t = c^n_t + g^n_t + i^n_t + AC^n_t \]  \hspace{1cm} (19)

where \( y^n_t \) represents the output arising from operating the Cobb-Douglas technology described in equation (14), and \( i^n_t \) is the net investment (i.e. the net increment in the stock of capital) in the nontraded sector. The equilibrium condition for traded goods requires that:

\[ p^n_t (c^n_t + g^n_t + i^n_t + AC^n_t + m^n_t) + (g^n_t + c^n_t) = y^x_t + y^x_t + \Omega + b_t R - b_{t+1} \]  \hspace{1cm} (20)

where \( y^x_t \) represents the domestic production of the non-oil, exportable goods, and \( i^x_t \) is the net increment in the stock of capital used in the exportable industry. The constraint indicates that sources and uses of internationally traded resources must be balanced in every period.

The country obtains international resources by producing and selling non-oil exportable goods; by exporting oil \( y^e \) at the price \( p^e_t \); by collecting financial income from international assets; and by adjusting its net asset position. On the other hand, the country uses international resources for private and public consumption and for the production of capital goods.

The competitive equilibrium of the economy is a sequence of consumption allocations of non-oil goods \( \{c^m_t, c^n_t, c^x_t\}_{s=1}^\infty \); a sequence of leisure-labor allocations, \( \{l^c_t, \ell^x_t, \ell^n_t\}_{s=1}^\infty \); a sequence of importable inputs to produce oil \( \{e^m_s\}_{s=1}^\infty \); a sequence of bonds and capital stocks, \( \{b^s_t, b^n_t, k^n_t, k^x_t\}_{s=1}^\infty \); a sequence of prices, \( \{p^e_t, p^m_t, p^n_t, p^x_t, W^n_t, W^x_t, R^n_t, R^x_t\}_{s=1}^\infty \); a sequence of oil transfers \( \{T^e_t\}_{s=1}^\infty \); a sequence of realizations of the shocks \( \{E_s\}_{s=1}^\infty \); and initial conditions for the stocks of bonds and capital, \( b^n_0, b^s_0 \) and \( k^n_0, k^x_0 \), such that:

1) The sequences of consumption and labor-leisure allocations, along with the sequences of
bonds and capital stocks, solve the constrained, infinite horizon, dynamic optimization problem of the household, given the initial conditions.

2) The sequence of allocations of capital and labor solve the profit maximization problem of the final-goods producing firms, and as a result of the optimization, firms obtain no profits.

3) The sequence of oil prices determines the allocation of the input to produce oil.

4) The government budget constraint holds with equality.

5) Markets for traded and nontraded goods clear and so do the labor and capital markets.

3.6 Calibration

The foregoing model of an oil-exporting small open economy is calibrated to two Andean economies, Peru and Colombia. In the non-stochastic steady state, some model macroeconomic ratios are set to mimic the corresponding ratios in these two economies, whereas the source of stochasticity seeks to represent the dynamics of their productivity and terms of trade shocks.

In both calibrations below, overall output and several relative prices are normalized to unity, starting with the price of the exportable good, which is chosen to be the \( \text{numeraire} \). The following are also normalized to one: a) the relative prices of importable and non traded goods; b) the price of oil; c) the use of labor in the production of exportable goods; and d) the GDP, defined as \( GDP = p^n y^n + y^x + \tilde{y}^e \), where \( \tilde{y}^e \) is the rent accruing to the owner of the natural resource \(( p^e_i y^e_i - p^m_i m^e_i )\).\(^{13}\)

Standard values in the RBC literature are used for some parameters: the marginal rate of intertemporal substitution \( 1/\sigma \), which is set to 0.5, and the elasticity of labor supply, which is defined by setting \( \omega_l = 1.455 \). As in Mendoza (1995), the elasticity of substitution between

\(^{13}\)By definition \( GDP = p^n (c^n + g^n + i^n + AC^n_i) + (y^x + y^c - c^x) - p^m (c^m + g^m + i^m + m^e + AC^m_i) \), where \(( y^x + y^c - c^x )\) is the exports demand.
traded and nontraded goods is set equal to 1.28 ($\eta = -0.218$); the rate of depreciation of exportable goods, $\delta_x = 0.1$, and the rate of depreciation of non-traded goods, $\delta^n = 0.05$.\textsuperscript{14}

3.6.1 Peru

The calibration of the model to the Peruvian economy is performed by using National Accounts data for the 1990-2011 period, compiled by the Peruvian Central Bank (available at \textit{http://www.bcrp.gob.pe/}). The frequency of the data is annual so that one period in the model corresponds to a calendar year in the Peruvian economy. Calibration ratios and parameters are summarized in Table 2.

Starting with aggregate macroeconomic ratios, Figure 6 shows that, except for government consumption, the ratios to GDP of each component of aggregate demand have been quite volatile over time. On average, for the period 1990-2011, household consumption has been equal to 71.1% of GDP; private investment (igdp) to 17.5%; and government consumption 13.1%. As for the trade balance, the sample average of the import ratio (m) is equal to 17.9%, while the export ratio (x) is equal to 12.8%.

The calibration strategy matches the aforementioned aggregate demand ratios with the model's steady state ratios. We assume that investment in construction is devoted to the accumulation of capital in the non-traded goods sector; for the period 1990-2011, 56.2% of private investment was in construction. Given that the level of non-traded investment in steady state is equal to $\delta^n k^n$, following the normalization of GDP and of $p^n$, that implies that $k^n = 0.562 \frac{igdp}{p^n} (\delta^n p^n) = 1.96$. In a similar fashion, the stock of capital used in the exportable industry, $k^x = (1 - 0.562) \frac{igdp}{(\delta^x p^m)}$, which implies that $k^x = 0.76$.

The ratio of importable consumption goods to GDP is $c^m$, and it is calculated as $c^m = m - \delta^x k^x - m^c = 0.10$. According to national accounts data, 70.8% of GDP belongs to the non-traded goods industry (services, construction and commerce), $y^n$; 4.4% of GDP is mining ($y^e$); the remaining fraction of output is taken to be the exportable-goods output ($y^x = 24.8\%$).

\textsuperscript{14} Setting a lower depreciation rate in the non-traded goods industry follows the fact that housing and real estate goods in general are the main component of the capital stock in that sector, which tends to last longer that capital goods used in the tradable sector.
In the model, the external demand for exportable goods is equal to \( d^x = y^x + y^e - c^x \); then, given the observed ratios of \( d^x \), \( y^x \) and \( y^e \) the value of consumption of exportable goods is pinned down by making \( c^x = y^x + y^e - d^x = 0.17 \). With the values of \( c^x \), \( c^n p^m \) and 

\[
C = (c^x + c^n p^m + c^n p^n) / y, \]

the consumption of nontraded goods arises as \( c^n = [C y - (c^x + c^n p^m)] / p^n = 0.44 \).

Mining exports account for a 51.5\% of total exports, which are equal to 8.4 percentage points of GDP, and therefore, to mimic this ratio \( y^e = 0.084 \). According to equation (15), and the normalization of \( p^e \) and \( p^n \), the input requirement for mining production is equal to \( m^e = (y^e - \gamma^e) / p^m \). From the market equilibrium condition for non-traded goods (19), fiscal expenditures on non-traded goods is equal to \( g^n = y^n - c^n - \delta^n k^n = 0.13 \), and it is equal to the observed ratio of final government consumption to GDP.

Available data for the 2003-2011 period for public debt and interest payments indicate that the average implicit interest rate on public debt is equal to 5.3\%. Since international and domestic bonds are perfect substitutes, they share the same net interest rate \( R - 1 = 0.053 \). Workers’ remittances (\( \Omega \)) is 1.4\% of GDP for the period 1991-2011. The external asset position, \( b \), is obtained from the market clearing conditions for tradable goods in equation (20):

\[
b = [p^m (c^n + g^m + \delta^m k^m + m^e) + c^x - y^x - y^e - \Omega] / (R^* - 1)
\]

A recent IADB study by Casas (2012) shows that fiscal revenues from the mining sector are equivalent to 2.5 percentage points of GDP; therefore, \( I^e = 0.025 \). In Andean countries, most of the commodity fiscal revenue is collected throughout the tax system, in particular through income related taxes; thus, in order to calibrate the consumption and income tax rates arising from tax revenue collections from consumption and income, we subtract from the observed income-tax revenues to GDP 80\% of the observed \( I^e \) and the remaining 20\% is subtracted from consumption-tax revenues to GDP. According to the cited study, \( \varepsilon_4 \) is equal to 0.7; \( \varepsilon_3 \), in turn, is calibrated by using equation (16) which is required to match the observed value of \( I^e \). Using the historical data for income-tax revenue the effective income tax rate (\( \tau^I \)) must be equal to 4.5\%. Following the same procedure for the effective consumption-tax rate and using historical data for
consumption-tax revenues obtains $\tau^c = 0.101$. Then, in order to satisfy the government’s budget constraint, equation (17), $\bar{T}$ is equal to -0.01.

With the value of $\tau'\ell$ at hand, the steady state value of the Euler equation for bonds (11) implies that $\beta = 1/(1+(R-1)(1-\tau')) = 0.95$. Then, using the values of $\delta^i$, for $i = n, x$, and $y^i$ for $i = x, n$, and the normalization of $p^i, i = m, n$, it is possible to obtain the output parameters $\alpha^i$ ($i = x, n$) from the capital Euler equations (13) as follows:

$$\alpha_i = \frac{[R - 1 + \delta]k^i}{y^i}, i = n, x \quad (21)$$

Thus, $\alpha_n = 0.28$ and $\alpha_x = 0.47$. Equation (9) indicates that the wage rate must be the same in the non-traded and exportable goods industries, and profit maximization implies that the value of the marginal product of labor must be equal to its rental price: $p^i F^i_{\ell^i} = W^i, i = n, x$, where $F^i_{\ell^i}$ is the marginal product of labor in the $i$ industry. As $F^x_{\ell^x} = F^n_{\ell^n} p^n$, the amount of labor used in the non-traded industry arises as follows:

$$\ell^n = [(1-\alpha_n) y^n]/[(1-\alpha_x) y^x \ell^x] \quad (22)$$

with $\ell^x = 3.86$. According to this calibration, around 80% of the labor supply is used in the non-traded sector and the remaining in the exportable sector. The values of the scale parameters $A^i$ ($i = n, x$) follow:

$$A^i = y^i/[(\ell^i)^{1-\alpha_i} (k^i)^{\alpha_i}] \quad (23)$$

and $A^n$ and $A^x$ are equal to 0.22 and 0.28, respectively. Given the values of $A^n$, $\ell^n$, $\alpha_n$ and $k^n$, the value of the real wage in the non-traded sector, $W^n$ can be obtained as:

$$W^n = (1-\alpha_n) A^n (\ell^n)^{1-\alpha_x} (k^n)_{\alpha_x} \quad (24)$$

which solves for $W^n = 0.13$. Keep in mind that wage rates are the same in both productive sectors.

The values of $a$ and $\chi$ are pinned down from equations (8a) and (8b), respectively. Given the values of the tax rates, ($\tau'$ and $\tau^c$), the values of $W^n$ and $\ell^n$ and $\ell^x$, and the value of
it is possible to get the value of $\omega^0$ from equation (9). The values of $\phi^i, i = n, x$ are chosen such that the volatilities of the model's investments in the exportable and non-traded goods sectors match the variability of investment data for machinery and equipment, and investment data in construction, respectively. The elasticity of $y^e$ with respect to $p^e$ ($\varepsilon_2$) was estimated by an OLS regression between $\ln(y^e_t)$ and $\ln(p^e_t)$; the chosen time series for $p^e$ was the metals' price index reported for the World Economic Outlook. The parameter ($\varepsilon_2$) is pinned down from equation (15) and is chosen to match the observed value of $y^e$. The parameter of the endogenous discount factor ($\psi$) is equal to $\ln \beta / \ln[1 + c_{i-1} - \omega_y / \omega_t (\ell^m_{i-1} + \ell^x_{i-1})^{\alpha}]$, which determines $\psi = -0.405$.

The parameters of the VAR(1) process of the sources of uncertainty were calibrated as follows. Due to the lack of enough sectoral data, we calculated the aggregate total factor productivity (TFP). Then, we multiplied the ratios $y^e_t / y_t$ and $y^x_t / y_t$ in order to get the time series for $A^e_t$ and $A^x_t$, respectively. The time series for the price of importable goods ($p^m$) were chosen so as to mimic the inverse of the observed terms of trade (reported by the Central Bank of Peru), previously adjusted for the price of metals ($p^e$) so that the obtained series of $p^m$ gets rid of the effect of $p^e$. There is no data available about terms of trade without $p^e$, but there is disaggregated terms of trade for imports and exports ($ToT_m$ and $ToT_x$, respectively). So, we regress the $ToT_x$ in levels with respect the main international commodity prices (oil, agricultural goods, fish and metals). Then, we recalculate a new $ToT_x$ (say $ToT_{x,1}$) using the coefficients of the OLS regression without $p^e$. Thus, $p^m_t$ was calculated as $ToT_m / ToT_{x,1}$. We applied an H-P filter to these series and their cyclical components were used to estimate the parameters of the VAR(1) process in equation (18). Only the coefficients statistically significant at 10% of confidence were taken to be different from zero\(^{15}\) (SEE Mendoza (1995) and Mendoza and

\(^{15}\)Other two VAR(1) models were estimated without finding significant differences in the results, one without any restriction on the coefficients and another restricting the non significant coefficients to be equal to zero. The statistical moments are very similar to the used specification, although they do not allow the model to mimic actual business cycles as the chosen model does, except for the correlation between GDP and the current account-to-GDP ratio.
Oviedo (2006) for cases of VAR(1) estimations adapted to the discrete state space estimations). For the Peruvian case, only the autocorrelation coefficients in the main diagonal of $\Gamma_1$ and the ones linking the two productivity shocks were different from zero.

### 3.6.2 Colombia

The calibration strategy applied to Peru is replicated to Colombia. Only the data used for the calibration procedure and some arising caveats are discussed here. Table 3 shows the variables and calibrated parameters. Data sources for the Colombian calibration are the Central Bank of Colombia (Banco de la Republica, www.banrep.gov.co); World Economic Outlook (WEO); and the National Direction of Statistics (www.dane.gov.co).

Whereas the influence of natural resources on the Peruvian economy is a phenomenon of relatively large age, the influence of oil in the external and fiscal accounts is only significant in recent years in Colombia. For instance, only for the period 2004-2011 the ratio oil exports to GDP is higher than the ratio oil's added value to GDP (see Figure 8). If the data shows that oil exports ($y^e$ in the model) is much lower than the oil's GDP ($p^e y^e - p^m m^e$ in the model), then the model is not strictly designed for that case because it would require adding a domestic consumption of oil. Things are different if we restrict ourselves to the period 2004-2011, in which the ratio oil exports to GDP are aligned with the design of the model. That is the period used in the calibration.

Having a very short of period of time where the importance of oil in the Colombian economy is in line with the model, it makes no sense to compute business cycle statistics as we did with in the Peruvian case nor to compare the model and the actual economy's impulse-response functions to shocks to the price of oil.

### 4. Benchmark Model: Public Debt as a Random Walk

The general equilibrium framework used to model the interactions between the shocks hitting the economy and the two productive sectors has two special features. One is the arising dichotomy between public debt and the rest of the economy. When the government keeps its outlays and tax rates constant while its fiscal revenues are affected by fluctuations of both the tax bases and the
value of the natural resource, the stock of government debt is determined by the rest of the economy but has not feedback effect on the economy. This dichotomy leads to the other special feature of the model which is that public debt follows a "random walk", a feature of public debt early remarked by Barro (1979) in a model of a closed economy.

When public debt follows a random walk, and it lacks a well-defined limiting distribution, its dynamics is determined by the initial stock of public debt and the sequence of shocks hitting the economy. If the starting conditions include a moderate to highly indebted government, it is likely that all sequences of shocks lead to an exploding level of debt. On the contrary, even under an extremely adverse sequence of shocks, the debt would tend to disappear and the government pile up international financial assets if the starting conditions include a small level of government indebtedness. This is why another way of characterizing the random walk behavior of public debt is by saying that the stock of public debt in any point in time depends on the initial conditions.

Alternatively, it is possible to show that for reasonable starting conditions of public debt, there are sequences of realizations of the shocks that will push the debt up without limits and another sequences of shocks that will bring it down without limits (becoming a large public asset). Figure 9 illustrates this feature of the model when the stock of public debt starts at its steady state value.  

Operationally, the dichotomy in the model implies that the government budget constraint could be excluded from the solution to the model without modifying the optimal allocations. Later, the dynamics of the fiscal balance and public debt could be recovered from the model's solution. This is what it is done in the remaining of this section.

It merits noticing that when some conditions are imposed on the behavior of the government, e.g. foreseeing the public sector to react to exogenous shocks (like variable taxes and/or expenditures), the model becomes stable and public debt becomes stationary. This issue will be covered in section 5, where public debt sustainability is analyzed.

---

16Once the model was solved by linearization around its steady state, it was simulated 1,000 times using taking the steady state of the model as its initial condition and the dynamics of all variables is stored for the analysis.
4.1 Benchmark Simulations in the Peruvian Calibration

The benchmark formulation of the model with debt as a random walk is solved to compare its business cycle statistics to moments of the Peruvian data. Overall, the model mimics the qualitative features of actual business cycles. From a quantitative perspective, however, Table 4 shows that the model finds it hard to produce statistical moments that closely match the data.

Starting with overall macroeconomic aggregates, consistently with the data, output displays a high correlation with consumption and total investment in the model, although they still fall short of the value of these correlations in the Peruvian data. It is particularly important to remark that the actual high correlations are much larger than those observed in other emerging market countries, and hints at possible data issues or other facts of the Peruvian economy that are not being captured by the model of Section 3.

On the other hand, model and data are quite close when it comes to compare the variances and serial correlations of the three main aggregate variables (GDP, consumption and investment). The model variance and autocorrelation of labor also mimics the data although their correlations with GDP seem to be too high in the model.

Moving to sectoral outputs, the model explain better the dynamics of the nontradable sector than that of the tradable sector. The variance, serial correlation and correlation with GDP for the non-traded goods sector are much closer to the moments of the data. Finally, the correlations between the international price of metals, \( p^e \), and all of the aforementioned macroeconomic and sectoral aggregates are sign consistent with the data.

The behavior of the model external sector also mimics qualitatively well what happened in Peru. Quantitatively, again, the statistical moments seems to exceed the Peruvian numbers. Oil and total exports in the model display a higher variability than in the data. The autocorrelation and correlation with GDP of oil exports (compared with total exports) are much closer to the observed moments. Total imports outperforms total exports, both in sign and in the value of the statistical moments. In fact, the variance and autocorrelation of imports in the model are closer to those of the data. And the current account (in percentages of GDP) displays results quite satisfactory in terms of its variance and correlation with output.
The model's real exchange rate (RER) seems not to fit quite well the data, except for its correlation with output. However, partially vindicating the model, notice that by design, the used measure of the RER is not affected by changes in the price of oil (the country's abundant natural resource) because of the assumption that oil is not consumed domestically. Although the signs of the correlations (except with $p^f$) are correct, the autocorrelation of the RER is quite higher in the model than in the data. Its variance and the correlation with GDP are closer to the data. The terms of trade (ToT) display a satisfactory model performance by closely matching the observed data moments. But on the other hand, whereas both the correlation between current account and GDP, and between the RER with $p^f$ are small in the data, the model predicts that these correlations are high and negative.

Given the assumption that tax rates and fiscal expenditures are fixed in the benchmark formulation of the model, government debt displays an explosive trajectory, and so only the statistics of fiscal revenues are computed for the government. Furthermore, as data on oil revenues are not available for a significant number of years in Peru, only total revenues and tax revenues are entering into the model-data comparison. Overall, model statistics are quite similar to data moments, particularly for total fiscal revenues. Discrepancies in the correlations between total revenues and $p^f$ may be explained by the significant amount of commodity related revenues that are obtained through the tax-system, rather than through royalties or the own exploitation of the natural resource that is assumed in the model.

### 4.2 Impulse - Response Functions

To understand the connection between the value of the natural resource owned by the government and the dynamics of the model, impulse and response functions of a negative shock to the oil price are shown in Figures 10 and 11. The shock in the model is equal to a one percent fall in the price of oil with respect to its steady-state value.\textsuperscript{17} Overall, an oil price fall produces the expected dynamics in an economy where the sovereign and the external accounts depend, to

\textsuperscript{17}In order to isolate this from other shocks, all but the shock to the price of oil are shut down in this exercise.
a large extent, on resources coming from the exploitation of a natural resource. A fall in the price of the exportable commodity produces a GDP fall of smaller magnitude and an intermediate fall in aggregate consumption. Non-traded consumption as well as tradable consumption \((c^x\) and \(c^m\)) fall.

Notice that the model displays a Dutch-disease effect: when the price of the natural resource rises, it appreciates the real exchange rate, leading to a fall in the country competitiveness and thus to a decrease in the output of the domestic manufacturing sector. The price and output of nontradable goods decrease due to the lower demand for these goods (-0.007% and -0.004%, respectively); on the other hand, output in the exportable sector increases. As the value of the marginal product of labor in the nontradable sector decreases, labor moves to the exportable sector until wage rates across sectors are equalized; thus \(\ell^n\) and \(\ell^x\) show a variation of -0.005% and 0.012%, respectively. Aggregate investment increases by 0.003% and the increment is explained by the investment in the exportable sector, which increases by 0.012% as the representative household finds it more profitable to move capital from the nontradable to the exportable sector.

The fall in the terms of trade (ToT) is equal 0.34% and it has a negative effect on the fiscal balance.\(^{18}\) Total fiscal revenues and the primary surplus decrease by 0.13% and 1.2%, respectively.\(^{19}\) While tax related revenues fall by 0.014%, oil fiscal revenues show a drop of 0.7%.

The current account balance-to-GDP ratio and the trade balance decrease by 0.034% and 2.07%, respectively, when both balances are measured at the constant, steady state prices. The net asset position \((b)\) evolves according to the current account balance measured at current (not constant) relative prices (equation 20), and it decreases by 0.61%.\(^{20}\) Commodity related exports

\(^{18}\)The ToT was constructed as the ratio of two Laspeyre indexes, namely index for exports divided by index for imports. The quantities for the base year in both indexes were the steady state value for exports and imports, respectively.

\(^{19}\)Income tax revenues coming from taxing interest income earned on government bonds are excluded in these measures.

\(^{20}\)By definition: \(b_t = b_{t-1} + \text{current account}\)
fall by 1.87%, non-oil exports increase by 0.10%, and total exports fall by 0.39%. Finally, the decline in the trade balance is curbed by the fall in imports (-0.19%) that is due to falls in importable consumption goods and in the inputs required to produce oil ($m^e$).

5. **Fiscal Stabilization and Fiscal Sustainability**

The benchmark formulation of the model mimics well the dynamics of the Peruvian economy during the last 20 years (1990-2011), specially its main aggregate and sectoral variables. The model can also be a useful tool to study the dynamics of small open economies characterized by the ownership of an omnipresent natural resource whose price variations impact private sector's production, investment, and consumption decisions as well as the fiscal balance and the future debt repayment obligations.

The model has a clear prediction about public debt. If tax rates and expenditures remain fixed, public debt is a random walk whose value a few years in the future could rise to levels that are unreasonable high compared to the levels tolerated by financial markets in practice (see Section 4). In the actual world, when public debt surpasses, say the value of one GDP or even a lower mark, warning alarms go off in financial markets and interest rates start to increase to reflect the building risks of default. Depending on how far these risks are allowed to grow, a set of fiscal responses are set on. When fiscal imbalances have grown large and public debt becomes definitely unsustainable, public debt default, high inflation, and deep fiscal reforms are all feasible elements of standard combos.

Governments that seek to avoid these extreme measures undergo smaller adjustments in fiscal policy before entering into the dangerous territories of solvency, increasing tax rates and reducing expenditures when public debt enters into dangerous territories and reverting the policy when public debt falls, thus guaranteeing that the stock of public debt remains checked. As it is well documented by Gavin and Perotti (1997a,b), this policy has been common among Latin American countries although it seems not to be exclusive to the region, as discussed by Talvi and Vegh (2005). The model of Section 3 is modified below to take into account possible fiscal reaction functions that would avoid extreme values of public debt and would assure fiscal sustainability. Two policies are considered in turn. First, tax rates are raised (decreased)
according to the extent that public debt departs from its steady state’s value. And second, while keeping the same tax rates, now the reaction to the departures of debt from the steady state is through the adjustment of government expenditures. In both cases, we do not assess the optimality of the policies but take cyclical macroeconomic smoothing as a policy goal in itself.

A consequence of these policies is that public debt is now stationary in the model and the early mentioned dichotomy disappears (see Section 4), reintroducing other feedback effects between the private and public sectors. For instance, a shock that reduces fiscal revenues and raises the stock of public debt will make the government to increase the tax rates which in turn will affect investment and consumption decisions, feeding back into the public budget. In this context, the convenience of having a general equilibrium model becomes even more evident.

Given the high correlation between ToTs and fiscal variables, and the high correlation between \( p^e \) and ToTs (see Section 2); we analyze fiscal adjustments policies that take place when an adverse shock to \( p^e \) occurs for a prolonged period of time. For the following sections, the Peruvian calibration of the model is revised to adopt the statistics observed during the 2000-2010 period. Even when, in general, there are no big changes in the level of macroeconomic ratios, it has been considered as desirable to capture (a) the sharp decline in the Peruvian debt ratio, which fell from 46.9% in 2003 to 21.2% in 2011, and (b) the significant increment of both imports and exports as shares of GDP. For the case of public debt, we took the average of the period 2007-2011 equal to 25% of GDP.

### 5.1 Avoiding Debt Hikes by Increasing Taxes

Under this reformulation of the model, income- and consumption-tax rates respond to prevent public debt to grow without limits. In particular we adopt the following adjustment in tax rates:

\[
\tau_i^t = \tau_{ss} \exp[\psi_2(b_{ss}^g - b_{i}^g)], \quad i = I, C
\]

which indicates that tax rates adjust according to the gap between the steady-state and actual levels of public debt, and where we consider two possible values of \( \psi_2 : \psi_2 = 1 \) and \( \psi_2 = 3 \). Notice that this kind of cyclical fiscal policy belongs to the core fiscal strategy adopted in Europe following the financial crisis started in 2008-09.

Figure 12 compares the reactions of the economy to the discussed terms-of-trade shock in the benchmark formulation of the model and in the tax-adjustment versions. We focus on GDP,
and some fiscal and external variables. As explained in the precedent section, without any fiscal response to the shock, the level of economic activity falls (see the fall in GDP). This implies that although government expenditures are fixed, their ratio to GDP increases. Although not shown in the figure, consumption, investment, and income all fall. Fiscal revenues fall due to the endogenous components of tax revenues (i.e. the shrinking of tax bases) and due to the direct impact that oil has on public finances, i.e., both oil price and production fall having both direct effects through equation . Falling fiscal revenues and constant levels of expenditures can only be financed by issuing more debt. And once the growing level of public debt starts to add interest expenses to the budget, the stock of debt will grow forever. Once $p^r$ ceases to fall and returns to its steady-state level, public debt ($-b^r$) has increased from 25.0% to 34.3% of GDP and will never become under check.

When fiscal policy includes endogenous tax rates, the upward adjustment in the tax rates adds further recessive pressures. Present consumption becomes more expensive and investment falls as the household adjust to a lower after-tax return on capital. Overall, GDP falls more in this economy. The fall is across the border because it affects both the tradable and nontradable sectors. Whereas in the baseline case, GDP is around 1.9% lower than its steady-state value while the fall in $p^r$ persists, under the endogenous tax regime, GDP reaches its minimum (-3.0% and -3.4% for $\psi_2 = 1$ and $\psi_2 = 3$, respectively) in the last year of the fall of $p^r$ (see Figure 12).

With the reactive taxes in place, two opposite effects on tax revenues arise. The already present endogenous response coming from the lower domestic absorption makes tax revenues to fall but the higher tax rates push revenues up. The latter effect dominates as can be seen by the increment in the tax revenue-to-GDP ratio. As for the stock of public debt, under the endogenous tax regime, the increment goes from 25.0% to 30.6% of GDP for $\psi_2 = 1$ (and to 28.4% for $\psi_2 = 3$), 3.8 percentage points less than in the baseline case. More important, public debt will eventually return to its steady state value.

The trade balance and the current account both worsen less under the endogenous tax-rates regime. While in the fixed tax rates model the trade balance surplus decreases by 1.5 percentage points in the first year of the fall in $p^r$, in the other regime the trade balance deficit worsens by 1.2 percentage points for both $\psi_2 = 1$ and $\psi_2 = 3$. The better performance of the
external accounts is explained by the reduction of the domestic demand. In fact, consumption of tradable goods falls more in the model with variable tax rates (due to the increase in $\tau^e$) reducing imports and increasing non-traditional exports ($y^e - c^e$). The current account behaves as the trade balance; in the artificial economy with fixed tax rates, in the first year that $p^e$ falls, it shows a deficit of -1.6% of GDP; while in the model with variable tax rates, the current account deficit is -1.15% of GDP, for both $\varphi_2$ equal to 1 and 3.

5.2 Cutting Expenditures

Instead of adjusting tax rates, policymakers could avoid debt hikes by cutting down government expenditures. Under this variant of the model the cyclical policy is aimed at keeping public debt checked by making expenditures in the non-traded sector to follow the reaction function that follows:

$$g^n_i = g^n_s \exp \psi_2 (b^e_i - b^e_s)$$

Here, public expenditures adjust according to the size of the gap between the steady state and the current level of public debt. We conduct exercises with the same two values of $\psi_2$ as in the precedent section. We start with the first parameter value and argue that results do not change considerably when the second value is used instead. Figure 13 shows that the paths of GDP and public debt under the expenditures adjustment regime is highly similar to the paths seen under the tax adjustment regime. By construction, the sectoral components of absorption are different, since the adjustment in expenditures happens in the non tradable market, depreciating the real exchange rate. Thus while the production of nontradable goods falls, the production of exportable goods increases. Consumption of non-traded goods increases as well, while the demand for traded goods (exportable and importable) falls. Overall, the income effect prevails and total consumption falls.

However, GDP falls less when the policy consists in cutting government expenditures than when it consists in raising tax rates. Output decreases by around 2.2% under both values of $\psi_2$ compared with the decrease in more than 3.0% in the case of raising tax-rates. In the last year of the low $p^e$ the public debt ratio to GDP is 29.6% and 27.2% for the cases of $\psi_2 = 1$ and 3, respectively. Tax revenues are lower following the diminishing income and consumption tax.
bases. Government expenditures display a decreasing trajectory due to the fiscal rule chosen to stabilize the level of public debt.

External accounts display the same behavior as in the tax rates adjustment regime. As the income effect takes place, the domestic demand falls more in the adjustment case. In the first year of the fall in $p^e$, the current account displays a deficit of 1.23% and 1.21% of GDP for $\psi_2=1$ and 3, respectively. The trade balance displays a surplus of around 1.3% of GDP in both cases.

### 5.3 Fiscal Sustainability: The Colombian Case

As in the precedent exercises, the analysis starts by exploring how public debt reacts when the artificial economy is calibrated to Colombia and is subject to a terms-of-trade shock. The cases with and without fiscal policy stabilization are considered in turn. As in the Peruvian case, the starting point is a fall in the price of oil of 30%. The dynamics of public debt in three cases are analyzed. Namely, the benchmark case, in which the government keeps constant the tax-rates and public expenditures; and the two other cases of fiscal policy stabilization, one in which the government adjusts the tax-rates and the other in which it decreases fiscal expenditures in order to stabilize the public debt to GDP ratio. Given that the economic behavior of the model is identical to the one observed in the Peruvian calibration, the main numerical results for Colombia are shown for a value of $\psi_2=1$ according to equations (25) and (26).

Figures 14 and 15 shows the results for the Colombian case. In the benchmark model, for the whole period in which $p^e$ is lower than its steady state value, the debt-to-GDP ratio increases from 34.0% to 39.9% while output falls 1.7%. Under a fiscal stabilization regime where the adjustment takes place through the tax rates, the debt ratio only increases by 2.8 percentage points (from 34.0% to 36.8%) and output falls by 2.4%. When the adjustment goes through endogenous public expenditures, the debt ratio increases by only 1.5 percentage points and output falls by 2.1%.

All in all, considering the Peruvian and Colombian calibrations, results indicate that a regime where fiscal sustainability is coming from the management of tax rates produces higher output and consumption costs than a regime where sustainability is reached by adjusting government expenditures. The latter mechanisms also produces a lower variability of public debt.
and debt levels remain more under control. These results are in line with the conclusions of an ongoing research by Andrian et al. (2012) who show that when government expenditures are unproductive, growth rates and consumption suffer less when fiscal deficits are controlled by cutting expenditures than when they are controlled by adjusting the tax rates.

6. Countercyclical Fiscal Policy

Despite the empirical findings reported by Gavin and Perotti (1997a,b) and Talvi Vegh (2005) indicating that developing countries follow procyclical fiscal policies, it seems that Peru took countercyclical fiscal actions to wind off the effects of the recession that affected Latin America in 2009. In this section, a modified version of the model of Section 3 helps to explore the macroeconomic and fiscal effects of a countercyclical fiscal policy that resembles the policy applied in Peru in 2009. As the model is incorporating the actual Peruvian fiscal policies, its fiscal and macroeconomic predictions are closer to what has been observed in the country's national accounts. In 2009, the international financial crisis caused a fall in the Latin American terms of trade and a considerable fall in the external demand for the region's exportable goods. Just in 2009, metals and oil prices suffered annual drops of 19.2% and 36.3%, respectively.21 The model is therefore forced to take a tantamount drop in $p^e$, while keeping constant the other sources of stochasticity. In particular, it is assumed that after three years of running on its steady state, the model faces a fall in $p^e$ equal to 22.7%, which is of the same size as the fall in the cyclical component of the price of minerals reported in footnote 20. During the periods before as well as during the periods after the shock occurs, it is assumed that the price of the natural resource remains at its steady state level.

According to the data, the Peruvian policy response to the 2009’s shock can be characterized by a substantial increment in government expenditures which, on annual basis, grew by 18.1%, according to national account statistics.22 Although hard to distinguish between

______________________________

21The fall in the cyclical components of these prices were 22.7% (for metals) and 24.7% (for oil), when these components are computed at annual frequency

22This fiscal policy continued during 2010.
policy responses and the endogenous consequences of the shock, the increment in government expenditures was accompanied by a fall in tax revenues of 11.8% and a stagnant GDP that only grew by 0.86% in the shock year.

As the working model has detrended variables, model business cycle statistics with those arising from HP filtered data. In particular, this section considers data on Peruvian output, external accounts, fiscal balance, and debt-to-GDP ratio and it computes the reaction of these variables during the shock year 2009. As shown in Table 5, detrended output fell by almost 0.6%, the trade balance improved by more than 1.6%, the fiscal balance worsened by 2.3%, and the public debt ratio increased by more than 3 percentage points.23

First, if the government in the model does not react to the shock and keeps the pre-shock taxes and expenditures unchanged, the model predicts a fall in output that exceeds by more than 240% the observed fall in the actual, detrended, Peruvian GDP (see Benchmark Model, column (b), of Table 5). The fall in the trade balance is well matched by the model but the fiscal balance and the public debt ratio both fall short of what has been observed in Peru. Finally, the model has some hard time explaining the reaction of the current account. That is, although the current account display a small surplus (detrended) of 0.18% of GDP, the model predicts a deficit of 0.26% of GDP.

If we are interested in using the model to study fiscal policy in Andean countries, it is worth to explore further why the model fails to reasonably mimic what has been observed in the Peruvian fiscal variables. Our strategy consists in forcing the model to adopt the same increment in nontradable government expenditures observed in Peru, equal to a 7.3% with respect to its trend value. In column (c) of Table 5 we report the results, which now show that the fiscal deficit and public debt ratio are much closer to the data, something that also happens with the fall in GDP. The model however fails to explain the observed reversal of the current account.

In column (d) of the table we report the results obtained after forcing the model to complement with a tax rate policy the observed 7.3% increment in government expenditures

23 As for the foreign accounts, only the trade balance but not the current account is reported at constant prices. This unparallel reporting occurs because while the source of the trade balance data is the national accounts, the source of the current account data is the central bank, which reports the dollar value of the variable. Data on debt-to-GDP ratios have not been detrended because the available time series are too short.
observed in 2009. In particular, we want to replicate the consumption tax policy observed in Peru during that year. As shown by Casas (2012), a large fraction of the tax revenues coming from natural resources in Andean countries proceeds from direct taxes. As we are interested in the consumption tax policy, we remove from the fiscal revenue data the fiscal revenues coming from natural resources. We take the result to represent $\log(\tau^c_t c_t)$ which we decompose into $\log(\tau^c_t)$ and $\log(c_t)$; we use the available data on the former and the latter of these three logarithms to construct data on $\tau^c$. Results indicate that $\tau^c$ fell by 7.9% in 2009. As column (d) shows, when the model is forced to adopt the observed increment in $g^n$ and the 7.9% fall in $\tau^c$, results are even closer to the data, confirming that the model is a useful tool to study fiscal policy in countries where fiscal revenues include the sovereign exploitation of a source of a natural resource.

### 6.1 Countercyclical Fiscal Policy: The Case of Colombia

In the shock year 2009, detrended Colombian data shows that the price of oil fell by 23%, output remained constant, the trade balance improved by 1%, the fiscal balance worsened by 1.1%, and the public debt-to-GDP ratio increased by more than 5 percentage points.

Table 6 reports the results for the Colombian case. Qualitatively, the same results as in the Peruvian case arise, although the model fails to produce quantitative results that are as close to the data as in the Peruvian case. As in the Peruvian case, if the government does not react to the shock and keeps the pre-shock tax rates and expenditures unchanged, the model predicts a fall in output of 1.4% (see Benchmark Model, column (b), of Table 6). The observed fall in the trade balance is not matched by the model and the fiscal balance and public debt ratio both fall short of what has been observed in Colombia.

However, when the government applies counter-cyclical fiscal policies, by increasing the level of expenditures (1.4% for detrended data) and decreasing the consumption-tax rate (5.7% 

---

24 We are unsure about the sources of the fall in $\tau^c$ since there was no official change in the consumption tax rate in Peru. We suspect that the fall is due to tax evasion, which in the model cannot be distinguished from a legislated reduction in $\tau^c$. What we are sure about is that the effective consumption tax rate fell as reported in the text.
for detrended data), the model shows results closer to what has been observed in the fiscal and output data. Thus, if the government only increases the level of non-traded expenditures (column (c) of Table 6), GDP falls by 1.3%, and the fiscal deficit doubles (-0.8%) compared to the benchmark case, while public debt increases (1.7 percentage points). However, the model again has a hard time in trying to explain external accounts, showing a larger gap between data and model's prediction than in the Peruvian case.

Finally, the case where an expansionary expenditures policy is accompanied by a reduction in the consumption tax rate is considered. As in the Peruvian exercise, output becomes closer to what has been observed in the data (-0.1%); additionally, the fiscal balance worsens by 1% and public debt rises by 1.74 percentage points. External variables are still far from reproducing the data. Despite this, it is fair to conclude this section by stating that the model performs very well when it comes to explain the dynamics of fiscal variables of Andean countries when a terms-of-trade shock hits their economies.

7. Concluding Remarks

By alerting about all direct and indirect effects that a macroeconomic shock has on an economy's fiscal balance, this paper has made it evident that simple, static, fiscal analysis may fall short to produce sound answers to policymakers interested in counting with solid stress tests of fiscal sustainability. An analysis that wants to take into consideration the multiple feedback effects existent between the fiscal balance and macroeconomic variables must be accompanied by a general equilibrium model. This paper has made the first steps in that regard, by building a tool capable of helping policymakers to soundly assess fiscal scenarios. This paper conducted exercises that illustrated the usefulness of the tool by evaluating alternative fiscal reactions, like adjusting expenditures and tax rates when public debt goes to dangerous territories.

This paper provides telling insight about the instruments that should be used in debt stabilization. When a prolonged ToT shock occurs and government expenditures are unproductive, fiscal adjustment should be done by cutting expenditures rather than increasing tax-rates, given that the loss in terms of output and consumption is lower by reducing expenditures.
General Equilibrium models are useful tools for analyzing and simulating counter-cyclical policies in countries where the government relies heavily in fiscal revenues related to commodities exports. In particular, in the Peruvian case, the model has shown a satisfactory performance in a country with a long history of related commodity fiscal revenue.

Further research should be conducted to take into account other types of shocks, like financial shocks, and to incorporate other elements of actual economies in order to fine tune the model until it becomes a highly reliable mechanism of fiscal analysis.
References


Table 1: Average Real GDP Growth: The 1990's versus de 2000's; and Public Debt Ratios: 2003 versus present

<table>
<thead>
<tr>
<th></th>
<th>Bolivia</th>
<th>Colombia</th>
<th>Ecuador</th>
<th>Peru</th>
<th>Venezuela</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average GDP Growth Rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1991-2000</td>
<td>3.59</td>
<td>2.76</td>
<td>2.32</td>
<td>3.70</td>
<td>2.30</td>
</tr>
<tr>
<td>2001-2010</td>
<td>4.09</td>
<td>4.52</td>
<td>6.39</td>
<td>6.39</td>
<td>3.54</td>
</tr>
<tr>
<td><strong>Public Debt to GDP Ratio</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>89.5</td>
<td>53.4</td>
<td>51.1</td>
<td>47.1</td>
<td>47.2</td>
</tr>
<tr>
<td>2011</td>
<td>33.9</td>
<td>42.9</td>
<td>22.7</td>
<td>21.7</td>
<td>32.7</td>
</tr>
</tbody>
</table>

*Source: Authors calculations based on ECLAC*

Table 2: Model Calibration to Peruvian Data: Parameters and Steady-State Value of Key Variables (Annual Frequency)

<table>
<thead>
<tr>
<th></th>
<th>Preferences</th>
<th>Technology</th>
<th>Fiscal Policy</th>
<th>Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>β</td>
<td>0.95</td>
<td>δ^x</td>
<td>0.10</td>
</tr>
<tr>
<td>-0.218</td>
<td>η</td>
<td>-0.405</td>
<td>δ^0</td>
<td>0.05</td>
</tr>
<tr>
<td>-0.405</td>
<td>ψ</td>
<td>1.455</td>
<td>α_x</td>
<td>0.47</td>
</tr>
<tr>
<td>0.021</td>
<td>ω^0</td>
<td>0.28</td>
<td>α_n</td>
<td>0.28</td>
</tr>
<tr>
<td>0.62</td>
<td>a</td>
<td>0.22</td>
<td>A^x</td>
<td>0.28</td>
</tr>
<tr>
<td>2.00</td>
<td>σ</td>
<td>7.29</td>
<td>b^x</td>
<td>0.032</td>
</tr>
<tr>
<td>0.17</td>
<td>c^x</td>
<td>1.14</td>
<td>φ^x</td>
<td>0.14</td>
</tr>
<tr>
<td>0.10</td>
<td>c^m</td>
<td>0.76</td>
<td>φ^n</td>
<td>0.76</td>
</tr>
<tr>
<td>0.44</td>
<td>c^n</td>
<td>1.96</td>
<td>k^x</td>
<td>1.96</td>
</tr>
<tr>
<td>k^x</td>
<td>k^n</td>
<td>1.00</td>
<td>e^1_r</td>
<td>1.00</td>
</tr>
<tr>
<td>k^n</td>
<td>e^0_r</td>
<td>3.87</td>
<td>e^0_r</td>
<td>3.87</td>
</tr>
</tbody>
</table>

*Source: Authors calculations.*
Table 3: Model Calibration to Colombian Data: Parameters and Steady-State Value of Key Variables (Annual Frequency)

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Technology</th>
<th>Fiscal Policy</th>
<th>Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>$\delta^x$</td>
<td>$\tau_r$</td>
<td>$p^e$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$\delta^n$</td>
<td>$\tau^i$</td>
<td>$p^m$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>$\alpha_x$</td>
<td>$\tau^i$</td>
<td>$p^n$</td>
</tr>
<tr>
<td>$\omega^0$</td>
<td>$\alpha_n$</td>
<td>$g^n$</td>
<td>$p^x$</td>
</tr>
<tr>
<td>$\omega^1$</td>
<td>0.0064</td>
<td>$A^x$</td>
<td>$b^g$</td>
</tr>
<tr>
<td>$a$</td>
<td>0.53</td>
<td>$A^n$</td>
<td>$I^e$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.00</td>
<td>$\phi^x$</td>
<td>$r$</td>
</tr>
<tr>
<td>$c^x$</td>
<td>0.12</td>
<td>$\phi^n$</td>
<td>$\epsilon_i^1$</td>
</tr>
<tr>
<td>$c^m$</td>
<td>0.10</td>
<td>$k^x$</td>
<td>$\epsilon_i^0$</td>
</tr>
<tr>
<td>$c^n$</td>
<td>0.44</td>
<td>$k^n$</td>
<td>$T^e$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k^x$</td>
<td>$\epsilon_i^1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k^n$</td>
<td>$\epsilon_i^0$</td>
</tr>
</tbody>
</table>

Source: Authors calculations

Table 4: Peru: statistical moments - Benchmark simulations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_{x_t,x_{t-1}}$</td>
<td>$\sigma_x$</td>
</tr>
<tr>
<td><strong>Domestic Demand and Production</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP (y)</td>
<td>0.68</td>
<td>0.039</td>
</tr>
<tr>
<td>Consumption (c)</td>
<td>0.71</td>
<td>0.034</td>
</tr>
<tr>
<td>Investment ($i^x + i^n$)</td>
<td>0.58</td>
<td>0.150</td>
</tr>
<tr>
<td>Hours ($h^x + h^n$)</td>
<td>0.64</td>
<td>0.037</td>
</tr>
<tr>
<td>Exportable production (y^x)</td>
<td>0.41</td>
<td>0.037</td>
</tr>
<tr>
<td>Non-traded production (y^n)</td>
<td>0.75</td>
<td>0.045</td>
</tr>
<tr>
<td><strong>External variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exports ($y^x - c^x + y^e$)</td>
<td>0.37</td>
<td>0.040</td>
</tr>
<tr>
<td>Imports ($c^m + i^x + AC^x$)</td>
<td>0.51</td>
<td>0.114</td>
</tr>
<tr>
<td>RER</td>
<td>0.09</td>
<td>0.032</td>
</tr>
<tr>
<td>ToT</td>
<td>0.47</td>
<td>0.083</td>
</tr>
<tr>
<td>Current Account / GDP</td>
<td>0.21</td>
<td>0.017</td>
</tr>
<tr>
<td>Oil Exports ($y^e$)</td>
<td>0.40</td>
<td>0.079</td>
</tr>
<tr>
<td><strong>Fiscal variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax Revenue</td>
<td>0.64</td>
<td>0.104</td>
</tr>
<tr>
<td>Total Fiscal Revenue</td>
<td>0.61</td>
<td>0.110</td>
</tr>
</tbody>
</table>

Source: Authors calculations based on official data and model’s simulations

Notes: Data moments calculated from de-trended data using HP-filter ($\lambda=100$).
$\sigma_x$ calculated in percentage of mean of x
### Table 5: Peru: Data and Model Reactions to a Terms-of-Trade Shock

<table>
<thead>
<tr>
<th></th>
<th>Data (a)</th>
<th>Benchmark Model (b)</th>
<th>Adj. in Expend. (c)</th>
<th>Adj. in Taxes and Expend. (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>-0.0059</td>
<td>-0.0142</td>
<td>-0.0134</td>
<td>-0.0075</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>0.0162</td>
<td>0.0031</td>
<td>0.0014</td>
<td>0.0067</td>
</tr>
<tr>
<td>CA/GDP</td>
<td>0.0018</td>
<td>-0.0026</td>
<td>-0.0010</td>
<td>-0.004</td>
</tr>
<tr>
<td>Fiscal Balance</td>
<td>-0.0239</td>
<td>-0.0058</td>
<td>-0.0146</td>
<td>-0.0195</td>
</tr>
<tr>
<td>Public Debt / GDP</td>
<td>0.0321</td>
<td>0.0135</td>
<td>0.0221</td>
<td>0.0253</td>
</tr>
</tbody>
</table>

*Source: Authors calculations*

*Note: Data moments correspond to HP detrended figures, except for the debt-to-GDP ratio.*

### Table 6: Colombia: Data and Model Reactions to a Terms-of-Trade Shock

<table>
<thead>
<tr>
<th></th>
<th>Data (a)</th>
<th>Benchmark Model (b)</th>
<th>Adj. in Expend. (c)</th>
<th>Adj. in Taxes and Expend. (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.0000</td>
<td>-0.0132</td>
<td>-0.0127</td>
<td>-0.0090</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>0.0102</td>
<td>-0.0075</td>
<td>-0.0079</td>
<td>-0.0084</td>
</tr>
<tr>
<td>CA/GDP</td>
<td>0.0029</td>
<td>-0.0065</td>
<td>-0.0069</td>
<td>-0.0078</td>
</tr>
<tr>
<td>Fiscal Balance</td>
<td>-0.0117</td>
<td>-0.0044</td>
<td>-0.0078</td>
<td>-0.0100</td>
</tr>
<tr>
<td>Public Debt / GDP</td>
<td>0.0500</td>
<td>0.0138</td>
<td>0.0166</td>
<td>0.0174</td>
</tr>
</tbody>
</table>

*Source: Authors calculations*

*Note: Data moments correspond to HP detrended figures, except for the debt-to-GDP ratio.*
Figure 1: Terms of Trade of LAC7 Countries (index 1990 = 100)

Source: Authors calculations based on WEO.

Figure 2: Terms of Trade and Fiscal Revenues for Andean Countries (indices 2000 = 100)

Source: Authors calculations based on Ministry of Finance of each country.
Note: Each index is an average of own countries’ indices. Countries: Bolivia, Colombia, Ecuador and Peru.
Figure 3: Terms of Trade and Public Debt for Andean Countries (indices 2000 = 100)

Source: Authors calculations based on ECLAC.
Note: Each index is an average of own countries’ indices. Countries: Bolivia, Colombia, Ecuador and Peru.

Figure 4: Overall Fiscal Balance vs Non-Commodity Fiscal Balance as percentage of GDP for Andean Country (Average 2010-2011)

Source: Authors calculations based on Ministry of Finance of each country.
Figure 5: ToT and Commodities’ Prices (Indices 2000 = 100)

Source: Authors calculations based on WEO.
Note: Each index is an average of own countries’ indices. Countries: Bolivia, Colombia, Ecuador and Peru.

Figure 6: Peruvian Aggregate Demand Ratios (% GDP)

Source: Central Bank of Peru.
Figure 7: Colombian Aggregate Demand Ratios (% GDP)

Source: Central Bank of Colombia

Figure 8: Colombian Oil Ratios (% GDP)

Source: Central Bank of Colombia
Figure 9: Public Debt as Random Walk

Source: Authors calculations

Figure 10: Impulse Response Functions to a shock in $p^e$

Source: Authors calculations

Note: percentage of steady state values.
Figure 11: Impulse Response Functions to a shock in $p^e$-cont.

Source: Authors calculations

Note: percentage of steady state values.
Figure 12: Peruvian Fiscal Sustainability: Increasing Taxes

Source: Authors calculations
Note: Variables in levels.
Figure 13: Peruvian Fiscal Sustainability: Cutting Expenditures

Source: Authors calculations

Note: Variables in levels.
Figure 14: Colombian Fiscal Sustainability: Increasing Taxes

Source: Authors calculations
Note: Variables in levels.
Figure 15: Colombian Fiscal Sustainability: Cutting Expenditures

Source: Authors calculations
Note: Variables in levels.