Template for External Sustainability Assessment

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Abstract[*]

This manual explains in detail the external sustainability assessment approach following Castillo (2016), given by the stock-flow relationship among net external positions, non-income current account,[1] and real exchange rate. This approach consists of determining the non-income current account over GDP that would stabilize a benchmark net foreign asset (NFA) position over the medium-term, and a "gap" by comparing the NFA-stabilizing non-income current account over GDP with its value expected to prevail over the medium term. The text develops an External Sustainability Template and provides guidance in the customization of empirical assessments by addressing three methodologies: Standard Approach, Endogenous Net Foreign Assets Dynamics Approach, and Fan-Charts Approach.

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[1] Hereafter, non-income current account balance plus net capital transfers will be cited as *non-income current account balance.*

1. Introduction

The external sustainability assessment (ESA) comes in many forms, such as medium-term balance of payment projections, external debt scenario analysis, assessing reserves adequacy, external vulnerability, and intertemporal solvency constraint analysis.

Our analysis addresses the *stock-flow* relationship between the sustainability of a country's net external positions and non-income current account balance plus net capital transfers. The external sustainability approach becomes a benchmark against other sophisticated econometric approaches, because it relies on assumptions regarding the long-run growth rate of real GDP in local currency units (LCU), long-run real rates of return on assets and liabilities, and a real exchange rate adjustment.[2]

Following Castillo (2016), we have developed a template to address an External Sustainability Assessment which provides guidance in the customization endeavors for empirical assessments by addressing three methodologies: Standard Approach, Endogenous Net Foreign Assets Dynamics Approach, and Fan-Charts Approach. This template offers considerable flexibility for implementing a variety of assumptions for each approach. Next, we describe each of the approaches in more detail.

The Standard Approach consists of determining the non-income current account balance[3] plus net capital transfers over GDP that would stabilize the net foreign asset (NFA) position over GDP at the particular benchmark level shown, which may be arbitrarily chosen by taking the latest year with data available (analysis of other benchmarks might also be of interest). The benchmark may be modified on the basis of regional averages or other criteria, however, for a select group of economies with extremely high net external liabilities, low net external liabilities, or that are exporters of non-renewable resources.

The second step compares the WEO-projected non-income current account balance over GDP (assuming real exchange rates that are constant at their early 2015 levels), with the NFA-stabilizing non-income current account balance over GDP, assuming no real exchange rate misalignments.

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[2] See Krugman (1999) and Goldfajn and Valdés (1999).
[3] Non-Income Current Account Balance plus Net Capital Transfers over GDP, henceforth called Non-Income Current Account Balance over GDP.

The Endogenous Net Foreign Assets Dynamics Approach depicts a scenario analysis of the future path of the NFA over GDP, subject to permanent shocks to each of its underlying variables of interest such as the following: long-run growth rate of real GDP in LCU, long-run real rates of return on assets and liabilities (including capital gains, if applicable), net current and capital transfers, change in real exchange rate, and foreign assets over GDP.

The Fan-Chart Approach holds that the expected future dynamic path of the NFA over GDP is uncertain and subject to temporary shocks. Fan-Chart Approach depicts the probability distribution of the NFA over GDP under a medium-term adjustment scenario by simulating $(n \times 1)$ shocks to each of its underlying determinants, where (n) is a large number. One deterministic and three statistical methodologies have been performed in the template to implement the Fan Chart Approach: i) External Forecasts, ii) VAR Approach, iii) External Forecasts with Correlated Errors and iv) Weighted Projections.[4]

The remainder of this document explains in detail each of the methodologies for addressing the External Sustainability Assessment.

2. External Sustainability Assessment Template

In this section, we provide step by step guidance in the implementation of the External Sustainability Template, which was programmed in Visual Basic, version 10 for Microsoft Excel 2010 [5] and addresses three approaches that evaluate the assessment of the sustainability of external positions: Standard Approach, Endogenous Net Foreign Assets Dynamics Approach, and Fan-Chart Approach. This template includes four spreadsheets: i) The Selected Series, ii) Standard Approach, iii) Endogenous Net Foreign Assets Dynamics, and iv) Fan Charts.

2.1 The "Selected Series" Spreadsheet

2.1.1 Defining the Data Intervals

The "Selected Series" spreadsheet is designed with the following objectives:

- Allowing the user to input data required for the external sustainability analysis.

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[4] Arizala et al. (2008) explain these methodologies in detail.
[5] If the template is not implemented in Excel Visual Basic 2010, it could generate software-related errors.

- Defining the range of analysis according to the country case study and user requirements.
- Validating the data entered into the spreadsheet.

The *"Selected Series" tab* features fields of data summarized as follows. Before performing the analysis; click on the *"0. Clear Data"* button. Then, set the data intervals in the range between B3 and D6 by entering the years in the following order: i) the initial and final years of available historical data, ii) last year of projections, and iii) the analysis horizon that stands for the long-run projections that the user may consider appropriate.
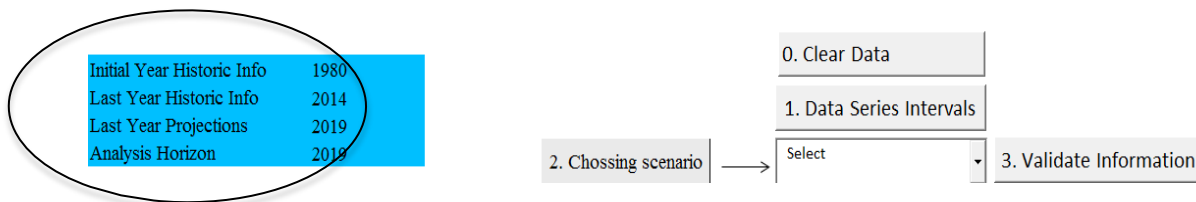
Next, we must click on the *"1. Data Series Intervals"* button to generate the fields in the spreadsheet (Column B). Once we have defined the data intervals, you may observe that each field is categorized by different colors. Historical data appear in white fields, short and medium-term projections appear in blue fields, and long-run projections appear in green fields, as shown in the figure below.[6] Additionally, the font colors of the variables to be entered in the "Selected Series" spreadsheet are blue and red, where the user manually enters the blue-colored variables, while the red-colored variables are created through formulas by the template.

Additionally, there are yellow-colored variables in the "Selected Series" spreadsheet (from cell V to cell AD), which are manually entered by the user and represent the decomposition of external assets and liabilities stocks. Having this information may be a valuable tool to identify whether assuming return differential in the ongoing exercises may be appropriate for a country. Otherwise, if that information is not available, the user may enter zeros instead.

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[6] Although it is not useful to make the External Sustainability Analysis beyond the medium term (five-year horizon) WEO projections, this template gives the flexibility to enter data beyond that period of time (i.e., the long-term projections). Thus, the template will fill in the long-term projections field with a constant value taking the latest medium term projection value for each variable of interest.

Year	U.S. Consumer Prices, end of period (2005=1)	Nominal GDP, in USD	Nominal GDP Growth Rate in LCU (%)	Nominal Exchange Rate (LCU per US $), End of Period	REER	Net Foreign Assets Positions	External Assets Stock	External Liabilities Stock	Current Account Balance	Net Current Transfers + Net Capital Transfers	Trade Balance (goods and services)	Income Balance	Exports Goods and Services	Imports Goods and Serivces	Private Remittances	Long-Run Total Real Rate of Return on Assets	Long-Run Total Real Rate of Return on Liabilities	Long-Run Real GDP Growth Rate in LCU	Long-Run Annual Change of NER
1990																			
1991																			
1992																			
1993																			
1994																			
1995																			
1996																			
1997																			
1998																			
1999																			
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2007																			
2008																			
2009																			
2010																			
2011																			
2012																			
2013																			

2.1.2 Assumptions on Medium-Term and Long-Term Projections

In general, if the variables were filled in up to the latest historical data field, the template would automatically assume that the medium-term projection field will be filled in by the latest historical values. However, it is also important to address the assumptions on the medium-term projections for some exceptions to be discussed in the table below.

Name of the Variables	Cells	Medium-Term Projections
U.S. Consumer Price Index	C9	WEO Projections.
Nominal GDP in USD	D9	
Real GDP Growth Rate in LCU (%)	E9	
Domestic Consumer Price Index	G9	
Net Foreign Assets	H9	Automatically extended: Cumulative medium-term forecasts of the current account balance from the World Economic Outlook (WEO).
BOP-derived Real Rate of Return on Assets	AZ9	Automatically extended assuming no real capital gains on assets: *Real yield on assets (Cells BB9)*
BOP-derived Real Rate of Return on Liabilities	BA9	Automatically extended assuming no real capital gains on liabilities: *Real yield on liabilities (Cells BC9)*

Consequently (if applicable) without exception, the long-term projections will be filled in with the latest value of the medium-term projections field.

2.1.3 Data Definition and Sources

Table 1 (see the Appendix) explains in detail the definition and suggested sources for each variable included in the spreadsheet. Some of the variables are compiled from the following sources:

5

- IMF's World Economic Outlook (WEO), which also contains estimations of medium-term (five-year horizon) forecasts.
- The gross external position data (at disaggregated and aggregated levels) can be compiled from the International Investment Position (IIP) Statistics of the IMF, Official Sources, and updates of the Lane and Milesi-Ferretti (2007) database.
- International Investment Position, International Financial Statistics Database: http://data.imf.org/?sk=5DABAFF2-C5AD-4D27-A175-1253419C02D1
- Investment Income Receipts and Payments may be collected from the International Financial Statistics (IFS) of the IMF: http://data.imf.org/?sk=5DABAFF2-C5AD-4D27-A175-1253419C02D1
- External Capital Outflows and Inflows (Financial Account) may be collected from the IMF's Balance of Payments Statistics Database (http://data.imf.org/?sk=7A51304B-6426-40C0-83DD-CA473CA1FD52) or Official Sources. As seen in the figure below, the lines that describe the external capital inflows and outflows are taken from the IMF's Balance of Payments Statistics Website. Total Capital Outflows may be calculated by adding up Direct Investment Assets, Portfolio Investment Assets, and Other Investment Assets. Likewise Total Capital Inflows may be calculated by adding up Direct Investment Liabilities, Portfolio Investment Liabilities, and Other Investment Liabilities.

Financial account (excludes reserves and related items)	4,526.5	-671.3	5,559.1	-2,604.9	-4,128.6	-3,492.8
˅ Direct investment, assets	1,488.0	1,054.8	890.0	1,789.1	252.4	196.1
Equity and investment fund shares	1,488.0	1,054.8	890.0	1,789.1	252.4	196.1
˅ Direct investment, liabilities	10,719.9	15,323.9	8,916.7	6,055.3	2,760.2	2,541.8
Equity and investment fund shares	8,120.4	12,203.8	9,699.7	5,881.3	2,760.2	2,541.8
Debt instruments	2,599.6	3,120.1	-783.0	174.0		
˅ Portfolio investment, assets	9.4	14.9	19.1	9.5	7.4	2.5
Equity and investment fund shares				-2.9	4.2	-0.7
Debt instruments	9.4	14.9	19.1	12.5	3.2	3.2
˅ Portfolio investment, liabilities	-1,891.5	-319.0	-212.0	5,161.8	898.4	634.7
Equity and investment fund shares	-174.0	876.2	461.8	-688.1	18.1	94.9
Debt instruments	-1,717.5	-1,195.2	-673.9	5,849.9	880.3	539.9
˅ Financial derivatives (other than reserves) and employee stock options	2,356.2	2,908.2	-32.4	-157.2	-12.2	-5.4
Fin. derivatives and employee stock options, liabilities	-2,356.2	-2,908.2	32.4	157.2	12.2	5.4
˅ Other investment, assets	15,963.5	8,429.0	10,704.9	4,615.3	1,450.3	2,149.3
Debt instruments	15,963.5	8,429.0	10,704.9	4,615.3	1,450.3	2,149.3
˅ Other investment, liabilities	6,462.1	-1,926.7	-2,682.2	-2,355.5	2,167.9	2,658.7
˅ Debt instruments	6,462.1	-1,926.7	-2,682.2	-2,355.5	2,167.9	2,658.7
Balance on current, capital, and financial account	-6,735.2	-450.6	-10,094.0	-3,213.7	397.3	1,409.4

The variables involved in the external sustainability analysis must be expressed as a percentage of GDP in US$. Consequently, the template calculates ratios over GDP in the cells between AI9 to AU9. These variables are red-colored in the spreadsheet and therefore calculated by the template.

2.1.4 Scenarios for Real Rates of Return on Assets and Liabilities

In the "Selected Series" Spreadsheet, cells R9 and S9 correspond to the Long-Run Total Real Rates of Return on Assets and Liabilities, respectively. These variables are red-colored in the spreadsheet, which means they are calculated by the template (and not entered manually) upon selecting an option from the rolling window, next to "*2. Choosing Scenario*".

7

Next, we explain the underlying assumptions to depict the real rates of return on assets and liabilities under each scenario addressed in the template.

- *The Baseline Scenario* may depict market indices as proxies for the real rate of return on assets and liabilities. For a further explanation of different alternative scenarios that might be considered beyond the market indices, see Box 2 in Castillo (2016). For example, the template describes that the long-run U.S. Treasury real yield may be a proxy for the real rate of return on assets (Cell AX9), and the long-run U.S. Treasury real yield plus EMBI yield for liabilities (Cell AY9).

- *The BOP Scenario* represents the BOP–derived Real Rates of Return on Assets (AZ9) and Liabilities (BA9), assuming Real Capital Gains/Loss during the historical period. For a further explanation about the BOP-derived return calculation, see Section 3 in Castillo (2016). They are automatically calculated by the template.

2.1.5 Validating Entered Data

In order to validate the entered data for each variable, we must click on the "*3. Validate Information*" button.

If all of the data are successfully validated, then the following dialog box will pop out:

				Input Data								
PPP Exchange Rate (LCU per US $)	REER	Net Foreign Assets	External Assets Stock					Income Balance	Exports Goods and Services	Imports Goods and Serivces	Private Remittances	Total Real Rate of Return on Assets
0	0	0	0	Microsoft Excel				0	0	0	0	0
0	0	0	0					0	0	0	0	0
0	0	0	0	Info Evaluation Finished Successfully				0	0	0	0	0
0	0	0	0					0	0	0	0	0
0	0	0	0					0	0	0	0	0
0	0	0	0			OK		0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

Otherwise, in the case of an error due to entering strings somewhere or off numbers, a dialog box will pop out indicating which year and variable the mistake has taken place in. Thus, after clicking OK, the template will display the problematic cell(s) in red. Then, the user must correct the error(s) and click on the "*3. Validate Information*" button again. This process must be carried out until the data are successfully validated.

				Input Data								
PPP Exchange Rate (LCU per US $)	REER	Net Foreign Assets	External Assets S					Income Balance	Exports Goods and Services	Imports Goods and Serivces	Private Remittances	Total Real Rate of Return on Assets
0	0	0	0	Microsoft Excel				0	0	0	0	0.00%
0	0	0	0					0	0	0	0	0.00%
0	0	0	0	Incompatible information entry in year:1997				0	0	0	0	0.00%
0	0	0	0					0	0	0	0	0.00%
0	0	0	0					0	0	0	0	0.00%
0	0	0	0			OK		0	0	0	0	0.00%
0	0	0	0	0	0	0	0	0	0	0	0	0.00%
0	0	A	0	0	0	0	0	0	0	0	0	0.00%
0	0	0	0	0	0	0	0	0	0	0	0	0.00%
0	0	0	0	0	0	0	0	0	0	0	0	0.00%
0	0	0	0	0	0	0	0	0	0	0	0	0.00%

2.1.6 Ancillary Tools

After having validated the data, the user may need to save, recover or delete country cases. Therefore, we have implemented ancillary tools for this purpose. Clicking on the *"Save Scenario"* button allows the user to save the whole dataset entered into the *"Selected Series"* spreadsheet. Conversely, clicking on the *"Delete Scenario"* button, allows the user to delete the scenario already saved. On the other hand, if the user has saved several country cases and wants to upload the scenario(s) afterwards, then she may want to use the *"Load info"* rolling window and select the desired scenario and load the data.

2.2 Approaches to Assessing External Sustainability

2.2.1 The Standard Approach: Spreadsheet

The Standard Approach is based on equation (2) of Castillo (2016), which calculates the required non-income current account plus net capital transfers over GDP (nca^s) that stabilizes *NFA* over GDP at a benchmark level[7] over the medium term, denoted as nfa^s, as follows:

$$nca^s = tb^s + ntr^s + kt^s = -\left(\frac{\left(r_s^{fl}-g_s^{LCU}\right)(1+\Delta X_s)}{(1+g_s^{LCU})} + \Delta X_s\right)nfa^s - \frac{\left(r_s^{fa}-r_s^{fl}\right)(1+\Delta X_s)}{(1+g_s^{LCU})}fa^s \quad (1)$$

where trade balance of goods plus services at benchmark level over GDP is denoted as (tb^s), net current transfers as (ntr^s), net capital transfers over GDP as (kt^s), and foreign assets over GDP as (fa^s). According to equation (1), the long-run real GDP growth rate in LCU, long-run real rates of return on assets and liabilities are denoted as ($\tilde{g}_s^{LCU}$), ($\tilde{r}_s^{fa}$), and ($\tilde{r}_s^{fl}$), respectively. Lastly, ($\Delta\tilde{X}_s$) denotes the "equilibrium" change in the real exchange rate consistent with the relative PPP condition at a benchmark level, where $\Delta\tilde{X}_s = 0$. For a further explanation, see Proposition 1 below.

Currently, the automatically chosen benchmark level for net foreign assets over GDP is the latest historical year for which data was entered into the "Selected Series" spreadsheet. Even though the benchmark has little normative content, it provides a useful perspective on whether projected trade balance at a current exchange rate is expected to affect the net external asset position over the medium term.

Proposition 1

Defining the long-run real exchange rate X as

$$X = \frac{eP^*}{P}$$

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[7] Variables at a benchmark level are denoted with superscript *s*, except for the real rates of return, annual change in the real exchange rate, and the real GDP growth rate in local currency units, which are denoted with subscripts *s* at benchmark levels.

10

*where e represents the long-run nominal exchange rate defined as the number of units of the domestic currency that can purchase a unit of a given US$, P*denotes the long-run U.S. price level, and P is the long-run domestic price level.*

We express the latter equation in percentage changes (denoted as Δ) as follows:

$$(1 + \Delta X) = \frac{(1 + \Delta e)(1 + \pi^{US})}{(1 + \pi^{DOM})}$$

where π^{DOM} and π^{US} represent the long-run domestic and U.S inflation rates, respectively. Therefore, we must ensure that the long-run change in the real exchange rate (ΔX) equals zero, which implies that the "PPP in relative terms" condition is satisfied:

$$\Delta e = \frac{(1 + \pi^{DOM})}{(1 + \pi^{US})} - 1$$

The "PPP in relative terms" condition says that inflation in the home country is given by the nominal exchange rate depreciation plus world inflation. Thus, when price levels are changing very rapidly these movements can dwarf all other factors, and then PPP provides a rather effective theory of exchange rate movements. ∎

2.2.1.1 Long-Run Variables Definition

In order to carry out the Standard Approach, we must set long-run variables such as: the long-run real GDP growth rate in LCU $\left(\tilde{g}_t^{LCU}\right)$, the long-run domestic $(\check{\pi}_t^{LCU})$ and U.S. $(\check{\pi}_t^{US})$ inflation rates, and long-run real rates of return on assets $\left(\tilde{r}_t^{fa}\right)$ and liabilities $\left(\tilde{r}_t^{fl}\right)$. Thus, we approximate the long-run variables as the *five-year* average rates[8], during the historical period, as defined below:

$$(1 + \widetilde{m}_t) = \left(\prod_{i=o}^{n-1}(1 + m_{t-i})\right)^{1/(n-1)} \tag{2}$$

where $m = \{g_t^{LCU}, \pi_t^{US}, \pi_t^{LCU}, r_s^{fa}, r_s^{fl}\}$ and $n = 5.$ The template calculates the long-run variables for the real GDP growth rate in local currency units (LCU) and the long-run domestic and U.S. inflation rates in cells T9, BH9, and BF9, respectively. In the "Selected Series" Spreadsheet, cells R9 and S9 correspond to the Long-Run Total Real Rates of Return on Assets and

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[8] Even though the five-year average percent change rate is an arbitrary measure and represents the medium term more than the long-term perspective. That window has been assumed mainly due to the lack of a long time series in some countries of the Latin American and the Caribbean region, but it might be changed without loss of generality.

Liabilities, respectively. Additionally, cell T9 corresponds to the Long-Run Real GDP Growth Rate.

The next figure collects the basic information needed to calculate the NFA-stabilizing non-income current account plus net capital transfers over GDP and its required adjustment to stabilize the external positions, consistent with the benchmark NFA over GDP.

Standard Approach	
NFA-Stabilizing Non-Income Current Account over GDP (Incl. Capital Transfers)	
Current NFA over GDP	-63.37%
Real Rate of Return on Liabilities	4.63%
Real Rate of Return on Assets	1.13%
Long-run real GDP growth rate in LCU	3.51%
Non-Income Current Account over GDP (Incl. Capital Transfers)	0.11%
NFA-Stabilizing Non-Income Current Account (Incl. Capital Transfers) over GDP	0.69%
Required Non-Income Current Account over GDP Adjustment	0.58%

According to the above image, the fields "Non-Income Current Account over GDP", "NFA-stabilizing Non-Income Current Account over GDP" and "Required Non-Income Current Account over GDP Adjustment to stabilize the external positions" will (automatically) vary depending on which options (explained below) are selected before undertaking the analysis.

2.2.1.2 Adjacent Scenarios

If the user would be interested in carrying out alternative scenarios beyond the "baseline" computed by the template, the user may input other assumptions for the endogenous variables used in the standard approach such as Long-Run Real GDP Growth Rate in LCU (Cells E48) and Long-Run Domestic Inflation Rate (Cell E51). This would affect the calculation of the "equilibrium" change in the real exchange rate consistent with the relative PPP condition. The Long-Run U.S. Inflation Rate remains unchanged (exogenous variable). Thus, the standard approach will be subject to different scenarios based on ad-hoc assumptions, by changing the Cells E48 and E51[9] of the "Standard Approach" sheet, as shown in the table below.

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[9] Once Cells E48 and E51are changed, the whole Standard Approach automatically changes.

Long-Run Variables Assumptions (in red may be changed by user)	
Long-Run Real GDP growth rate in LCU	3.51%
Equilibrium Annual Change of Nominal Exchange Rate	3.18%
Long-Run of U.S. Inflation rate	2.11%
Long-Run Domestic Inflation rate	5.36%
Equilibrium Annual Change of Real Exchange Rate	0.00%

2.2.1.3 Steps for Implementing the "Standard Approach" Analysis

1. Choosing the Non-Income Current Account over GDP measure to be compared to its NFA-stabilizing value.

1) Choosing the Non-Income Current Account over GDP measure to be adjusted	Select

After computing the NFA-stabilizing Non-Income Current Account Balance over GDP (nca^s) from equation (2), we measure the required non-income current account balance over *GDP* adjustment to stabilize the net external positions at the benchmark level, denoted as R_{adj}. For this purpose, we must first define the most appropriate *m*easure of Non-Income Current Account Balance *over GDP* $(\overline{nca})$ to calculate the required adjustment $\left(R_{adj}\right)$.

$$R_{adj} = nca^s - \overline{nca} \qquad (3)$$

This gap represents the required adjustment that may be needed to bring in line the most adequate measure of Non-Income Current Account Balance over GDP to its NFA-Stabilizing value. According to equation (3), in the template we must select a Non-Income Current Account Balance over GDP measure to be considered among the options below:

- *Historical Medium-Term* Non-Income Current Account Balance *over GDP*
- *Actual* Non-Income Current Account Balance *over GDP*
- *Medium-Term Projection* Non-Income Current Account Balance *over GDP*

Three measures of non-income Current Account over GDP	
Medium-Term non-income Current Account over GDP (incl. Capital Transfers)	1.39%
Actual non-income Current Account over GDP (incl Capital Transfers)	0.11%
Medium-Term Projection non-income Current Account over GDP (incl Capital Transfers)	1.99%

Those alternatives are listed on the *rolling window* next to "1. *Choosing the Trade Balance over GDP measure to be adjusted*". The selection of one (or more) alternative(s) must be borne out by each country's assumptions.

2. *Assuming or Not Real Rate of Return Differentials*

The alternatives "assuming or not Real Rate of Return Differentials are listed on the *rolling window* next to "2). Assuming or not Real Rate of Return Differentials".

2) Assuming return differential or not	Select Select ▾

Once we have set the Non-Income Current Account Balance *over GDP* measure, the template computes the NFA-stabilizing Non-Income Current Account Balance over GDP according to equation (2). Thus, if we want to assume real rate of return differentials (i.e., the expected rates of return on foreign assets are different from those paid out on foreign liabilities), this would imply that the second term of equation (2) does not cancel out.

For a detailed discussion about the determinants in explaining the Real Rate of Return Differentials, see Section 2.4 in Castillo (2016), which discusses how the composition and magnitude of the external portfolio of assets and liabilities, business cycles, exchange rates, and asset prices movements have an important role in explaining such return differentials.

2.2.1.4 Standard Approach: Sensitivity Analysis

Following equation (2), we have performed a *Sensitivity Analysis* in the *"Standard Approach"* spreadsheet for different values of Long-Run Real GDP growth rate in LCU ($\tilde{g}_s^{LCU}$) (Cell E6), and Long-Run Real Rate of Return on Liabilities $\left(\tilde{r}_s^{fl}\right)$ (Cell E4), which range is defined as an interval confidence given by $\gamma +/- jc\sigma$, where (γ) is denoted as the latest historical value for ($\tilde{g}_s^{LCU}$) and $\left(\tilde{r}_s^{fl}\right)$, ($\sigma$) is denoted as the standard deviation, (c) represents a constant, and (j) represents the deviation steps.[10]

In the header of the figure below, there are two fields to be set for ($\tilde{g}_s^{LCU}$) and $\left(\tilde{r}_s^{fl}\right)$. They may be changed by the user subject to her preferences and correspond to (j) deviation steps and a (c) constant for each variable. We explain each step below.

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[10] Deviation Steps determines the window length of the analysis for each variable.

Step 1 computes the NFA-Stabilizing Non-Income Current Account Balance over GDP for a defined range of values for $(\tilde{g}_s^{LCU})$ and $(\tilde{r}_s^{fl})$. Thus, in order to ensure that the economy is dynamically efficient, it must be satisfied that $\left(\tilde{r}_s^{fl} > \frac{(\tilde{g}_s^{LCU}-\Delta\tilde{e}_s)}{(1+\Delta\tilde{e}_s)}\right)$; conversely, when $\left(\tilde{r}_s^{fl} < \frac{(\tilde{g}_s^{LCU}-\Delta\tilde{e}_s)}{(1+\Delta\tilde{e}_s)}\right)$ the economy is dynamically inefficient. When $\left(\tilde{r}_s^{fl} = \frac{(\tilde{g}_s^{LCU}-\Delta\tilde{e}_s)}{(1+\Delta\tilde{e}_s)}\right)$, though, the economy is said to be at the Golden Rule level of capital accumulation.[11]

Step 1: NFA Stabilizing Non-Income Current Account (Incl. Capital Transfers)							
	Long-Run Real GDP Growth Rate in LCU						
	3.13%	3.26%	3.39%	3.51%	3.64%	3.77%	3.90%
Real Rate on Liabilities							
5.38%	1.38%	1.30%	1.22%	1.14%	1.06%	0.98%	0.90%
5.13%	1.23%	1.15%	1.07%	0.99%	0.91%	0.83%	0.75%
4.88%	1.08%	1.00%	0.92%	0.84%	0.76%	0.68%	0.60%
4.63%	0.93%	0.85%	0.77%	0.69%	0.61%	0.53%	0.45%
4.39%	0.77%	0.69%	0.61%	0.54%	0.46%	0.38%	0.30%
4.14%	0.62%	0.54%	0.46%	0.38%	0.30%	0.23%	0.15%
3.89%	0.47%	0.39%	0.31%	0.23%	0.15%	0.08%	NA

Step 2 computes the Required Non-Income Current Account Balance over GDP Adjustment (%) for a defined range of values for $\left(\tilde{g}_s^{LCU}\right)$ and $\left(\tilde{r}_s^{fl}\right)$ as in Step 1.

Step 2: Required Adjustment Non-Income Current Account (Incl. Capital Transfers) % GDP							
5.38%	1.27%	1.19%	1.11%	1.03%	0.95%	0.87%	0.79%
5.13%	1.12%	1.04%	0.96%	0.88%	0.80%	0.72%	0.64%
4.88%	0.97%	0.89%	0.81%	0.73%	0.65%	0.57%	0.49%
4.63%	0.81%	0.73%	0.66%	0.58%	0.50%	0.42%	0.34%
4.39%	0.66%	0.58%	0.50%	0.42%	0.35%	0.27%	0.19%
4.14%	0.51%	0.43%	0.35%	0.27%	0.19%	0.12%	0.04%
3.89%	0.36%	0.28%	0.20%	0.12%	0.04%	-0.04%	NA

2.2.1.5 Extra Simulation Panel

Additionally, we have performed a more flexible sensitivity analysis where the user may change the benchmark of each underlying variable (i.e., each red cell in the figure below) to compute multiple scenarios of the NFA-Stabilizing Non-Income Current Account Balance over GDP and Required Non-Income Current Account Balance over GDP Adjustment (%). Lastly, the *"All simulations"* field assesses the combined scenario of all changed underlying variables.

---

[11] The Golden-Rule level of capital therefore refers to the steady state capital that maximizes consumption per capita every period, which would in turn suggest an unbounded accumulation of NFA over GDP, which is not dynamically inefficient. For more details, see Ramsey (1928), Phelps (1961 and 1965), Cass (1965) and Diamond (1965).

Sensitivity Analysis per variable			
Variables of Interest	Values	NFA-Stabilizing Non-Income Current Account (Incl. Capital Transfers) over GDP	Requirement Adjustment % GDP
Long-Term Real Growth Rate in LCU	3.51%	0.69%	0.58%
Real Rate of Return on Liabilities	4.63%	0.68%	0.57%
NFA over GDP	-63.37%	0.69%	0.58%
Foreign Assets/GDP	33.70%	0.69%	0.58%
Equilibrium Long-Run Nominal Exchange Rate	3.18%	0.68%	0.57%
All simulations		0.68%	0.57%

2.2.2 Net Foreign Assets Dynamics Approach: Spreadsheet

The *Net Foreign Assets Dynamics* Approach undertakes a shock analysis based on the NFA over GDP dynamics, given by equation (4), that follows a first order autoregressive process in terms of underlying determinants: Trade Balance over GDP, Net Current Transfers over GDP and Net Capital Transfers over GDP, Real GDP Growth Rate in LCU, Real Rate of Return on Assets and Liabilities (inclusive Capital Gains when assumed BOP-derived returns), Annual Change of Real Exchange Rate, and Foreign Assets over GDP. Like the "Standard Approach," the "Net Foreign Assets Dynamics" Approach takes data from the "Selected Series" spreadsheet.

$$nfa_T = tb_T + ntr_T + kt_T + \left(\frac{\left(1+r_T^{fl}\right)(1+\Delta\tilde{X}_T)}{\left(1+g_T^{LCU}\right)}\right)nfa_{T-1} + \frac{\left(r_T^{fa}-r_T^{fl}\right)(1+\Delta\tilde{X}_T)}{\left(1+g_T^{LCU}\right)}fa_{T-1} \qquad (4)$$

where net foreign asset positions over GDP is denoted as (nfa_T), trade balance of goods plus services over GDP is denoted as (tb_T), net current transfers is denoted as (ntr_T), net capital transfers over GDP as (kt_T), and (fa_{T-1}) corresponds to the lagged foreign assets over GDP. According to equation (1), the real GDP growth rate in LCU, real rates of return on assets and liabilities are denoted as $\left(g_T^{LCU}\right)$, $\left(r_T^{fa}\right)$, and $\left(r_T^{fl}\right)$, respectively.

Without accounting for potential revaluation of the NFA over GDP in the medium-term could require a much larger real exchange adjustment to place the external position on a sustainable path in the baseline scenario. To operationalize a real exchange rate adjustment in the NFA dynamic (equation 4), we assume that current account's variables and GDP growth rates-WEO forecasts are calculated by holding the REER (X_T) constant at time T over the medium-term.

In the External Sustainability Assessment framework, we implement an adjustment of the real exchange rate misalignment by assuming a one-time jump in the REER from X_T to its

benchmark level X_T^* at time T which induces a gradual adjustment in the expected trade (exports and imports) flows during the forecasting horizon after T, in proportion to trade semi-elasticities. The intuition behind this assumption relies upon the steady-state conditions that a country must meet to ensure external sustainability over the long-run after closing the REER misalignment at the beginning (T) of the forecasting horizon.

For simplicity, in order to proxy the one-time jump in the REER from X_T to its benchmark level X_T^* at T, we take the real exchange rate misalignment at T from external sources (i.e. latest IMF's Article IV documents if available) or can be computed as the exchange rate adjustment required to close the gap between the observed non-interest current account deficit and the required current account balance that stabilizes the net foreign assets position given in equation (4).

In the context of WEO forecasts, the REER is assumed constant at time T over the forecast horizon; therefore the change in the REER does not have impact on the expected rates of return. However, expected returns could respond to the change in the REER through its impact on expected yields (e.g., if future interest payments are denominated in foreign currency); but reflecting this impact on future returns would require detailed information about the foreign currency composition of foreign assets and liabilities. Due to data shortcomings, we assume that the one-time jump in REER does not affect the path of expected returns in the baseline scenario.

In order to address the impact of a one-time jump adjustment in the real exchange misalignment at T on the expected trade balance in goods and services over the medium-term horizon, we follow Tokarick (2010),[12] who discusses in detail the implicit shortcomings and advantages of assuming a trade semi-elasticities partial equilibrium setup. Thus, it may be assumed that the relationship between real exchange rate marginal movements and trade balance over GDP (tb) in a small country depends on estimated trade semi-elasticities, as follows:

$$\frac{dtb}{dX/X} = \delta_{tb,reer} = (1 + \varepsilon^X \phi^E)S_X - (1 + \eta^M \phi^M)S_M) > 0$$

where S_X and S_M are the shares of exports and imports in GDP respectively; $\varepsilon^X > 0$ and $\eta^M < 0$ are the estimated export supply and import demand elasticities taken from Tokarick (2010) and defined with respect to changes in the real exchange rate (X), respectively. We define that, in a

---

[12] Box 1 tackles the conceptual issues about the underlying assumptions to estimate the Semi-Elasticity of Trade Balance over GDP to REER $(\delta_{tb,reer})$.

small country, an appreciation of the real exchange rate must worsen the trade balance (or alternatively, a depreciation of the real exchange rate will improve the trade balance), and ϕ^E and ϕ^M are the pass-through coefficients for export and import prices, respectively and lie between zero and one.[13] There is no consensus in the literature on values for these parameters. Frankel, Parsley, and Wei (2005) estimate that for developing countries and emerging markets, the pass-through coefficient is in the range of 0.66 to 0.77.

Therefore, equation (4) may be re-written in terms of the adjusted expected trade balance in goods and services (tb_T^*) over the medium-term horizon, measuring the impact of the one-time jump in the real exchange rate adjustment at T, as follows:

$$nfa_T = tb_T^* + ntr_T + kt_T + \left(\frac{\left(1+r_T^{fl}\right)\left(1+\Delta\tilde{X}_T\right)}{\left(1+g_T^{LCU}\right)} \right) nfa_{T-1} + \frac{\left(r_T^{fa}-r_T^{fl}\right)\left(1+\Delta\tilde{X}_T\right)}{\left(1+g_T^{LCU}\right)} fa_{T-1} \quad (5)$$

where tb_T^* is the adjusted trade balance over GDP after the one-time REER adjustment, and it is defined as follows:

$$tb_j^* = \begin{cases} \left(1 + \delta_{tb,reer}(X_T - X_T^*)/5\right)tb_j & \text{if } tb_T < 0 \\ \left(1 - \delta_{tb,reer}(X_T - X_T^*)/5\right)tb_j & \text{if } tb_T > 0 \end{cases} \quad \text{for } j = \{T+1 \ldots T+5\}$$

For example, if the real exchange rate of a country at $T\left(\tilde{X}_T\right)$ were initially overvalued (undervalued), the country would require to adjust the trade flows with a gradual depreciation (appreciation) that will improve (deteriorate) the trade balance over the medium term.

The values for export supply and import demand elasticities must be entered in the "Standard Approach" spreadsheet at cells (F,76-77), as shown in the next picture. By default, we have entered average values from Tokarick (2010), but they have to be changed subject to the country-study. Based on the entered export supply and import demand elasticities, the template calculates the semi elasticities of Trade Balance over GDP to Real Exchange Rate movements given by $\delta_{tb,reer}$.

Export Supply Semi Elasticity wrt REER > 0	1.1
Import Demand Semi Elasticity wrt REER < 0	-1.14

---

[13] Pass-through equal to 1 entails that pass-through is complete and changes in foreign prices are fully reflected in domestic prices.

We have additionally assumed values for $\left(\tilde{g}_T^{LCU}\right)$ after time T are defined as the WEO forecasts for real GDP growth rate, and we assume the latest historical value for $\left(\tilde{r}_T^{fa}\right)$, and $\left(\tilde{r}_T^{fl}\right)$ after time T.

2.2.2.1 Steps for Implementing the Net Foreign Assets Dynamic Approach

1. The first step to implement the NFA Dynamic Approach is entering an assumed real exchange misalignment (K_T)[14] at time T. Where a positive value stands for undervaluation: real exchange rate above its level implied by fundamentals at T. Conversely, a negative value stands for overvaluation: real exchange rate below its level implied by fundamentals at T.

<div align="center">

1. REER misalignment %
(+) initial undervaluated REER
(-) initial overvaluated REER

-11.5

</div>

2. The fifth term on the right-hand side of equation (5) stands for the returns differential between the real return on assets and liabilities. For a detailed discussion about the determinants in explaining the Real Rate of Return Differentials, see Section 2.4 in Castillo (2016). So the template offers the chance to select if returns differential is assumed or not by clicking on the rolling window below *"2. Choosing if returns differential is assumed or not"*.

<div align="center">

2. Choosing if return differential is assumed or not

Select ▾

</div>

3. There are *six* fields that correspond to each underlying variable to be shocked, according to equation (5). The comprehensive *shock analysis* can be enabled by clicking on each box at the left side of each variable in the figure below (column 1).[15] For the enabled variable(s), the user can select the shock duration[16] by defining the year ("Initial year", column 2) when the shock takes place until when the shock takes place until the end of the effect ("Final Year", column 3). Furthermore, we define the shock (ξ) on the right hand side of the figure below (column 5), as follows:

---

[14] Positive variation of the real exchange means undervaluation, and negative means overvaluation.
[15] When a box has not been clicked, then that variable is left out of the shock analysis.
[16] The shock duration must be set between *the last historical year -1* to the *last year of projections*.

$$\xi = \mu + c\sigma$$

where μ and σ correspond to the historical mean and standard deviation of the enabled variable, respectively. And the nuisance parameter c corresponds to a measure for shock size in terms of σ (column 4), which must be selected accordingly to the user's assumptions. For standardization purposes, it is proposed that all exercises assume a one standard deviation (negative) shock to each of the underlying variables, and for a combined scenario (all shocks simultaneously) to assume a 0.5 standard deviation (negative) shock, calculated by default in the template. It is worth pointing out that the user may consider alternative scenarios in order to facilitate comparison, based on own assumptions and considerations.

						Measured in std. dev.	Mean + std*size shock
☐ Long Run Real GDP Growth in LCU	Initial Year	2016	Final Year	2018	Size shock	-1	
☐ Real Rate of Return on Liabilities	Initial Year	2016	Final Year	2018	Size shock	1	
☐ Trade Balance over GDP	Initial Year	2016	Final Year	2018	Size shock	-1	
☐ Net Transfers over GDP	Initial Year	2016	Final Year	2018	Size shock	-1	
☐ Foreign Assets	Initial Year	2016	Final Year	2018	Size shock	-1	

Afterwards, we proceed to click on the *"3. Starting Analysis"* button, which will start the analysis for the enabled variables.

3. Starting Analysis

4. The figure below shows the Baseline Scenario (no shocks), where rows 1 to 5 and 8 represent the entered projected paths of all underlying variables of equation (6). The line 11 represents the simulated Net Foreign Assets over GDP based on equation (6). Line 6 corresponds to the assumed misalignment of the real exchange at time $T\left(\Delta \tilde{X}_T\right)$.

Baseline Scenario (No Shocks)	2014	2015	2016	2017	2018	2019	2020
Long Run Real GDP Growth (LCU)	4.10%	1.00%	2.00%	2.00%	2.00%	2.00%	2.00%
Real Rate of Return on Liabilities	9.35%	7.67%	7.67%	7.67%	7.67%	7.67%	7.67%
REER Misalignment (assumed to decline over the forecasting horizon)			-11.50%	-8.63%	-5.75%	-2.88%	0.00%
Trade Balance over GDP	-3.76%	-2.08%	-2.87%	-2.83%	-1.56%	-1.57%	-1.13%
Net Current and Capital Transfers (& Errors/Omissions) over GDP	7.52%	7.26%	6.04%	5.81%	5.64%	5.62%	5.52%
Adjusted Trade Balance over GDP			-2.87%	-2.72%	-1.50%	-1.51%	-1.09%
Delta Real Exchange Rate (one time jump REER adjustment)			-11.50%	0.00%	0.00%	0.00%	0.00%
Foreign Assets over GDP	41.12%	36.24%	34.51%	33.10%	31.56%	31.56%	31.56%
Real Rate of Return on Assets	1.99%	1.31%	1.31%	1.31%	1.31%	1.31%	1.31%
Simulated Baseline NFA over GDP (No-REER Adjustment)	-155.13%	-165.42%	-171.45%	-178.00%	-183.82%	-189.99%	-196.17%
Simulated Baseline NFA over GDP (REER Adjustment)	-155.13%	-165.42%	-151.37%	-156.70%	-161.27%	-166.13%	-170.94%

5. We now proceed to do the shock analysis for all underlying variables of the simulated NFA over GDP, according to equation (6), where each line in the figure below represents each shock scenario. The "Combined Scenario"[17] includes the whole set of simulated shocks, and the "Observed Net Foreign Assets over GDP" line represents the observed data and its projection based on the Cumulative Current Account Balance Forecast.

Analysis of Variable Shocks on NFA over GDP	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
Long Run Real GDP Growth in LCU	-59.03%	-63.37%	-57.84%	-53.13%	-49.51%	-47.15%	-45.46%	-43.75%	-42.02%	-40.27%	-38.5%	-36.7%
Real Rate of Return on Liabilities	-59.03%	-63.37%	-57.34%	-52.24%	-48.31%	-45.56%	-43.84%	-42.12%	-40.37%	-38.61%	-36.8%	-35.0%
Annual Change of the Real Exchange Rate	-59.03%	-63.37%	-57.41%	-51.92%	-46.82%	-41.83%	-40.08%	-38.32%	-36.53%	-34.73%	-32.9%	-31.1%
Trade Balance over GDP	-59.03%	-63.37%	-61.94%	-60.92%	-60.67%	-61.57%	-60.02%	-58.45%	-56.87%	-55.28%	-53.7%	-52.0%
Net Transfers over GDP	-59.03%	-63.37%	-68.46%	-74.09%	-81.00%	-89.70%	-88.44%	-87.17%	-85.88%	-84.58%	-83.3%	-81.9%
Foreign Assets	-59.03%	-63.37%	-57.35%	-52.27%	-48.36%	-45.61%	-43.90%	-42.17%	-40.43%	-38.67%	-37%	-35%
Simulation Combined Shock	-59.03%	-63.37%	-65.66%	-67.82%	-69.85%	-71.77%	-70.32%	-68.86%	-67.39%	-66%	-64%	-63%
Observed Net Foreing Assets over GDP (Extended CAB)	-59.03%	-63.37%	-68.42%	-73.05%	-77.08%	-80.34%	-82.94%	-82.94%	-82.94%	-82.94%	-82.94%	-82.94%

6. Once the Shock Analysis has been performed, we plot the projected shocked NFA over GDP paths for the selected scenarios in Figures 1 and 2. Figure 1 plots up to six scenarios, three of them by default: i) "Simulated Baseline NFA over GDP without REER Misalignment Adjustment", ii) "Simulated Baseline NFA over GDP with REER Misalignment one-time Adjustment, and iii) "Combined Scenario", plus three selected enabled variables by using the *"4. Choosing shocks to be plotted in Figure 1"* rolling windows for each shock as shown below.

4. Choosing shocks to be plotted. Figure 1

Shock 1	▼
Shock 2	▼
Shock 3	▼

---

[17] The "*combined scenario*" assumes half of each standard shock assumed in the analysis.
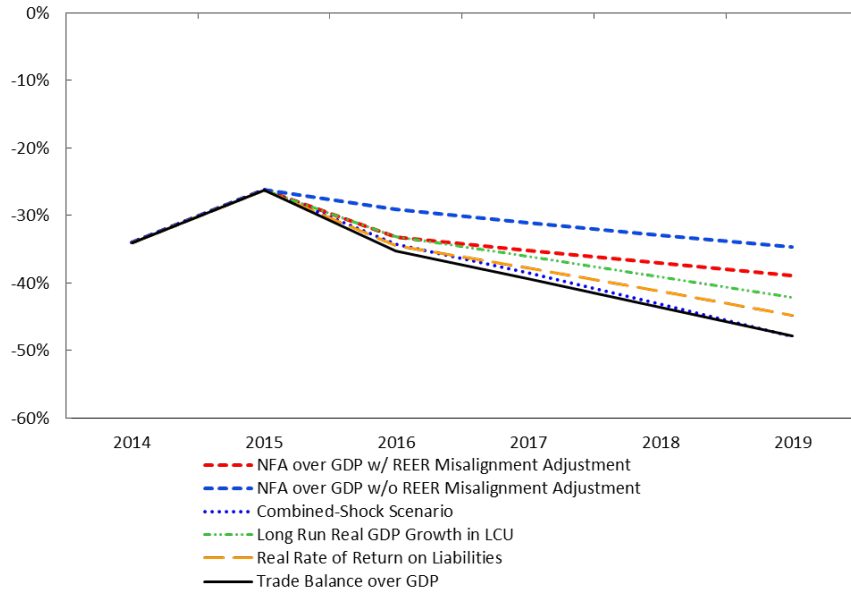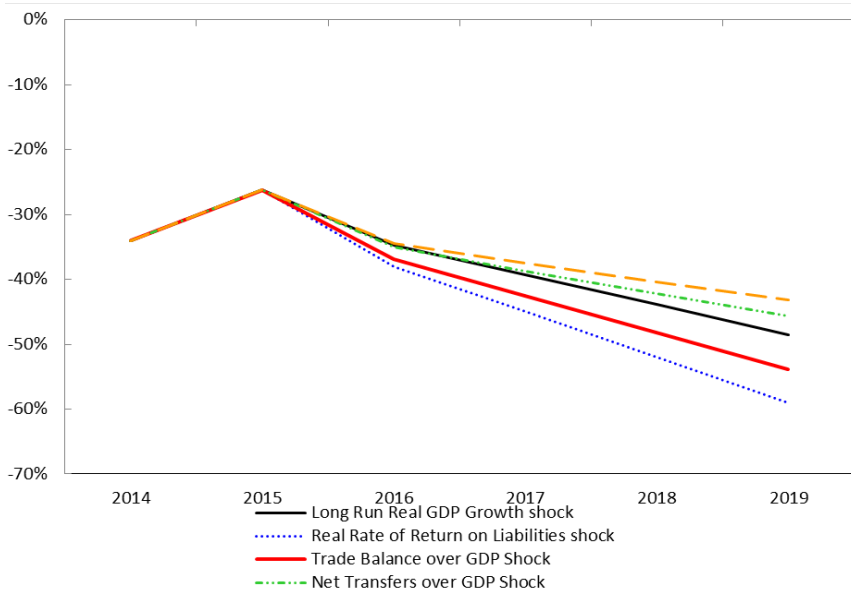
Figure 1. Simulated NFA over GDP

Legend:
- NFA over GDP w/ REER Misalignment Adjustment (red dashed)
- NFA over GDP w/o REER Misalignment Adjustment (blue dashed)
- Combined-Shock Scenario (blue dotted)
- Long Run Real GDP Growth in LCU (green dash-dot)
- Real Rate of Return on Liabilities (orange dashed)
- Trade Balance over GDP (black solid)

Figure 2 shows the *projected shocked NFA over GDP* paths for all the enabled variables in the shock analysis to obtain a better visualization of the *NFA over GDP* path after each shock.

Figure 2. Simulated Shocked NFA over GDP

Legend:
- Long Run Real GDP Growth shock (black solid)
- Real Rate of Return on Liabilities shock (blue dotted)
- Trade Balance over GDP Shock (red solid)
- Net Transfers over GDP Shock (green dash-dot)

7. Finally, we have a descriptive table that offers information about the shock pattern to each variable and its corresponding simulated projected *NFA over GDP* path evaluated in its corresponding shock over time.

Comparative Analysis of Shocks	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
Long Run Real GDP Growth - Shock	2.4%	3.5%	2.6%	2.6%	2.6%	2.6%	2.6%	2.6%	3.6%	3.6%	3.580%	3.580%
NFA / GDP Ratio	-50.9%	-46.7%	-60.4%	-57.8%	-55.7%	-54.1%	-53.0%	-51.9%	-50.3%	-48.6%	-47%	-45%
Real Rate of Return on Liabilities-Shock	5.0%	4.6%	4.6%	4.6%	4.6%	4.6%	4.6%	4.6%	4.6%	4.6%	4.63%	4.63%
NFA / GDP Ratio	-50.9%	-46.7%	-59.9%	-56.9%	-54.4%	-52.3%	-50.6%	-49.0%	-47.3%	-45.6%	-44%	-42%
Annual Change Real of the Exchange Rate-Shock		4.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	0.0%	0.0%	0.000%	0.00%
NFA / GDP Ratio	-50.9%	-46.7%	-59.1%	-55.1%	-51.3%	-47.4%	-43.4%	-39.6%	-37.8%	-36.0%	-34%	-32%
Trade Balance over GDP-Shock	-20.6%	-18.9%	-22.1%	-22.1%	-22.1%	-22.1%	-22.1%	-22.1%	-18.4%	-18.4%	-18%	-18%
NFA / GDP Ratio	-50.9%	-46.7%	-64.5%	-65.7%	-67.1%	-68.9%	-71.1%	-73.4%	-72.0%	-70.5%	-69%	-68%
Net Current and Capital Transfers over GDP-Shock	19.1%	19.0%	8.4%	8.4%	8.4%	8.4%	8.4%	8.4%	20.6%	20.6%	20.589%	20.59%
NFA / GDP Ratio	-50.9%	-46.7%	-71.0%	-79.1%	-88.1%	-98.0%	-109.0%	-120.1%	-119.2%	-118.2%	-117%	-116%
Foreign Assets over GDP-Shock	33.5%	33.7%	18.2%	18.2%	18.2%	18.2%	18.2%	18.2%	56.5%	56.5%	57%	57%
NFA / GDP Ratio	-50.9%	-46.7%	-59.9%	-56.9%	-54.4%	-52.4%	-50.7%	-49.1%	-47.4%	-45.7%	-44%	-42%

2.2.3 Fan-Charts Approach: Spreadsheet

The Fan-Chart Approach lays out that the expected future dynamic path of the Net Foreign Assets over GDP in equation (4) is uncertain and subject to temporary shocks. Fan Chart Approach depicts the probability distribution of the Net Foreign Assets over GDP under a medium-term adjustment scenario by simulating a large number of shocks to each of its underlying determinants. One deterministic and three statistical methodologies have been performed in the template to implement the Fan Chart approach: a) External Forecasts, b) VAR Approach, c) External Forecasts with Correlated Errors, and d) Weighted Projections. Arizala et al. (2008) explain these methodologies in detail; additionally, a brief description of each methodology is laid out in Box 1.

2.2.3.1 Steps for Implementing the Fan-Charts Approach

As we explained in the latter approach, the first step is choosing whether or not to assume returns differential according to equation (4), by selecting the preferred option in the below rolling window.

Return Differential

We now proceed to explain the *Fan Charts* methodologies covered in this template.

2.2.3.2 External Forecasts

After clicking on the *"External Forecast"* button, a dialog box pops up showing historical
standard deviation and the last observed data of each of the underlying variables of the NFA over
GDP identity given by equation (4).

☐ External Forecast

The dialog box below displays fields where the user may enter Exogenous Shocks to the
NFA over GDP projected simulation. After clicking on the *"Run"* button, another dialog box
will pop up indicating whether or not the simulation was successfully completed.

A Fan Chart will then project NFA over GDP path under the External Forecast scenario
as plotted below.

Fan-Chart: External Projections

Additionally, the template performs a Value at Risk Analysis of the Simulated NFA over GDP, which calculates the threshold-exceeding probability (in absolute values) of the NFA over GDP over time.

Value at Risk of the NFA / GDP				
Threshold of the NFA over GDP limit (X)				-70%
Prob(NFA/ GDP) < X				0.29

	Sensitivity Analysis of NFA over GDP			
Intervals	<-0.7	<-0.75	<-0.8	<-0.85
2015	0.32	0.17	0.07	0.03
2016	0.31	0.18	0.09	0.05
2017	0.28	0.15	0.08	0.03
2018	0.31	0.16	0.08	0.03
2019	0.26	0.13	0.05	0.02
Total	**0.29**	**0.16**	**0.07**	**0.03**

2.2.3.2 VAR Approach

Under this scenario, the first step is manually entering the Coefficient (Cells O4 to U11) and Variance-Covariance (Cells W4 to AC10) matrices estimated from a VAR(1) that comprises all of the underlying variables of the NFA over GDP identity given by equation (4): Trade Balance over GDP, Net Current and Capital Transfers over GDP, Real GDP Growth Rate in LCU, Real Rates of Return on Assets and Liabilities, Annual Change of Real Exchange Rate, and Foreign Assets over GDP.

List of Variables

Real GDP Growth (LCU)	g
Trade Balance over GDP	tb
Net Current and Capital Transfers over GDP	ntr
Real Rate of Return on Liabilities	rf
Real Rate of Return on Assets	ra
Foreign Assets over GDP	fa
IMF's Annual Change of the Real Exchange Rate	delta_x
Constant	c

Coefficient Matrix:

	g	tb	ntr	rf	ra	fa	delta_e
g							
tb							
ntr							
rf							
ra							
fa							
delta_x							
c							

2. VAR-COVAR Matrix

	g	tb	ntr	rf	ra	fa	delta_e
g							
tb							
ntr							
rf							
ra							
fa							
delta_e							

The user must enter the Coefficient and Variance-Covariance matrices as shown in the figure above. Note that in the figure above, there are fields colored in blue for the case that the user chooses *"No Returns Differential"*. In this case, Real Rate of Return on Assets and Foreign Assets over GDP must be excluded from the VAR estimation, causing the template to fill out the corresponding rows and columns with zeros. The next step is clicking on the "*VAR Approach*" button.

☐ VAR Approach

Afterwards, a dialog box pops up showing historical standard deviation and the last observed data of each of the underlying variables of the NFA over GDP identity given by equation (4). It also displays the estimated Coefficient and Variance-Covariance matrices already manually entered by the user and fields where the user may enter Exogenous Shocks to the projected NFA over GDP simulation.

After clicking on the *"Run"* button, another dialog box pops up indicating the simulation was successfully completed. Thereupon, a Fan Chart with the projected NFA over GDP path under the VAR scenario is plotted. The template also performs a Value at Risk Analysis of the simulated NFA over GDP, which calculates the threshold-exceeding probability (in absolute values) of the NFA over GDP over time.

VAR Approach

Coefficients from VAR(1)

	g	tb	ntr	rf	ra	fa_gdp	delta_x
g					0	0	
tb					0	0	
ntr					0	0	
rf					0	0	
ra	0	0	0	0	0	0	0
fa_gd	0	0	0	0	0	0	0
delta_x					0	0	
c					0	0	

	Standard Deviations	Beta	Last Observed Data
Long T Real GDP Growth LCU	1	0	0.035133
Trade Balance over GDP	1	0	-0.189388
Net Transfers over GDP	1	0	0.190495
Real Return on Liabilities	1	0	0.046347
Real Return on Assets	1	0	0.011318
Foreign Assets over GDP	1	0	0.337413
Long T Annual Change in Real Exchange Rate	1	0	0.010123

Run

Original VAR-COVAR Matrix

	g	tb	ntr	rf	ra	fa_gdp	delta_x
g	1	2	3	4	0	0	7
tb	8	9	10	11	0	0	14
ntr	15	16	17	18	0	0	21
rf	22	23	24	25	0	0	28
ra	0	0	0	0	0	0	0
fa_gdp	0	0	0	0	0	0	0
delta_x	43	44	45	46	0	0	49

Exogenous Shocks

year	Shock to NFA
2005	0
2006	0
2007	0
2008	0
2009	0
2010	0
2011	0
2012	0
2013	0
2014	0
2015	0
2016	0
2017	0
2018	0
2019	0

27

Box. 1. Fan Charts

The Fan-Chart tool incorporated in this template relies on historical annual data for: Trade Balance over GDP, Net Current and Capital Transfers over GDP, Long-Term Growth Rate of Real GDP, Real Rate of Return on Assets and Liabilities (inclusive Capital Gains), Annual Change of Real Exchange Rate, and Foreign Assets over GDP.

Historical data of each of the underlying variables of the NFA over GDP (discussed above) are used (in different ways) to generate the sample means and variance-covariance matrix that define the joint normal distribution for these variables.

Draws for each of those determinant variables from the joint normal distribution are used to generate shocks that are added to the baseline projections for each of the variables and fed into the NFA over GDP given by equation (4) to calculate a distribution of projected NFA over GDP.

By construction, based on the joint normal distribution of shocks, we generate a symmetrical fan chart. Therefore, it would be interesting to relax the normality assumption and address different effects on upside and downside risks.

The fan shape of the distribution over the projection period is due to the increased uncertainty over time. Accordingly, if simulations every year lead to adverse shocks (e.g., low real GDP growth), then the NFA over GDP will increase and evolve along a path in the upper bound of the fan chart distribution.

Conversely, if every year simulations lead to positive shocks (e.g., high real GDP growth), then the NFA over GDP will decline and evolve along a path in the lower bound of the fan chart distribution.

One deterministic and three statistical methodologies have been performed to implement this approach: a) External Forecasts, b) VAR Approach, c) External Forecasts with Correlated Errors and d) Weighted Projections. These methodologies are explained in detail in Arizala et al. (2008).

External Forecasts

This approach is carried out by generating shocks $iid\ e_t \sim N(0, \sigma_i^2)$ for each of the underlying variables of the NFA over GDP to then simulate $N = 1000$ repetitions of x_T^s of (τ) dimension, based on the following equation:

$$x_\tau^s = x_\tau^{ext} + e_t$$

where $\tau \in [t + 1, T]$, $\tau = 5$ corresponds to the longest forecast horizon assumed by the IMF, x_τ^{ext} corresponds to the assumed external projection of each underlying variable and σ_i^2 corresponds to its historical variance.

Afterwards, all x_τ^s are fed into the NFA over GDP given by equation (6) to calculate a distribution of projected NFA over GDP.

28

VAR Approach

This approach considers the following autoregressive model:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + e_t$$

where y_t is a vector of k dimension that comprises each of the k underlying x_τ^{VAR} variables of the NFA over GDP identity given by equation (4), the residual matrix e_t is normally distributed following $N(0, \Omega)$, where Ω represents the variance and covariance of the autoregressive model. In order to simulate the correlated k underlying variables paths (following the latter equation), we must use the Cholesky decomposition Ψ of the variance and covariance matrix Ω defined as:

$$\eta_t = \Psi v_t$$

with v_t normally distributed with mean 0 and variance 1. Once the k paths of dimension (τ, N) have been simulated, they are fed into the NFA over GDP given by equation (4) to calculate a distribution of projected NFA over GDP.

External Forecasts with Correlated Errors Approach

This approach follows the same External Forecast approach, but instead assuming correlated errors coming from the VAR Approach in the NFA over GDP simulation.

Weighted Projections

This is a weighted combination between External Forecasts with correlated errors and VAR Approach, following the equation below:

$$x_\tau^s = \beta x_\tau^{ext} + (1 - \beta) x_\tau^{VAR} + \eta_t$$

where x_τ^{VAR} corresponds to the simulation of each k underlying variable coming from the VAR model, and $\beta \in [0, 1]$. When $\beta = 1$, the latter equation collapses to the case of External Forecasts with correlated errors and when $\beta = 0$, it corresponds to the VAR Approach.

2.2.3.3 External Forecasts with Correlated Errors

After clicking on the *"External Forecast with Correlated Errors"* button, as shown below, a dialog box pops up showing historical standard deviation and the last observed data of each of the underlying variables of the *NFA over GDP* identity given by equation (4).

☐ External Forecast With Correlated Errors

It also displays the estimated variance-covariance matrix from VAR(1) and some other fields where the user may enter Exogenous Shocks to the projected NFA over GDP simulation. After clicking on the *"Run"* button, another dialog box pops up indicating the simulation was successfully completed.

Thereupon, a Fan Chart with the projected NFA over GDP path under the External Forecasts with correlated errors is plotted. The template also performs a Value at Risk Analysis of the Simulated NFA over GDP, which calculates the threshold-exceeding probability (in absolute values) of the NFA over GDP over time.

2.2.3.4 Weighted Projections

After clicking on the *"Weighted Projections"* button, as shown below,

☐ Weighted Projections

the following dialog box pops up, showing the historical standard deviation and the last observed data of each of the underlying variables of the NFA over GDP identity given by equation (4). It also displays the estimated coefficients and variance-covariance matrices from VAR(1) manually entered before and, additionally, a field to be manually modified β (beta). As explained in Box 1, when $\beta = 1$, the model collapses to the case with *"External Forecasts with correlated errors"*, and when $\beta = 0$, the model corresponds to the *"VAR Approach"*.

It also displays fields where the user may enter Exogenous Shocks to the projected NFA over GDP simulation. After clicking on the *"Run"* button, another dialog box pops up indicating the simulation was successfully completed.

Thereupon, a Fan Chart with the projected NFA over GDP path under the VAR scenario is plotted. The template also performs a Value at Risk Analysis of the Simulated NFA over GDP, which calculates the threshold-exceeding probability (in absolute values) of the NFA over GDP over time.

Bibliography

Allen, M. 2006. "Exchange Rates and Trade Balance Adjustment in Emerging Market Economies." Washington, DC, United States: International Monetary Fund.

Arizala, F. et al. 2008. "Debt Sustainability Fan Charts: Combining Multivariate Regression Analysis and External Forecasts." Washington, DC, United States: Inter-American Development Bank. Draft.

Cass, D. 1965. "Optimum Growth of Capital in an Aggregative Model Accumulation." *Review of Economic Studies* 32(3): 233-240.

Castillo, M.A. 2016. "External Sustainability Assessment: Theoretical Framework." Technical Note IDB-TN-2016. Washington, DC, United States: Inter-American Development Bank.

Devereux, M., and A. Sutherland. 2007. "Solving for Country Portfolios in Open Economy Macro Models." IMF Working Paper 07/284. Washington, DC, United States: International Monetary Fund.

Diamond, P. 1965. "National Debt in a Neoclassical Growth Model." *American Economic Review* 55(5): 1126-1150.

Eichengreen, B., and R. Hausmann. 1999. "Exchange Rates and Financial Fragility." NBER Working Paper 7418.

Evans, M., and V. Hnatkovska. 2007. "Financial Integration, Macroeconomic Volatility and Welfare." Journal of the European Economic Association 5(2-3): 500–508.

Frankel, J.A., D.C. Parsley, S-J. Wei. 2005. "Slow Passthrough Around the World: A New Import for Developing Countries?" NBER Working Paper 11199. Cambridge, United States: National Bureau of Economic Research.

Goldfajn, I., and R. Valdes. 1999. "The Aftermath of Appreciations." *Quarterly Journal of Economics* 114(1): 229-262.

Gourinchas, P-O., and H. Rey. 2007. "International Financial Adjustment." *Journal of Political Economy* 115(4): 665-703.

Krugman, P. 1999. "Balance Sheets, the Transfer Problem, and Financial Crises." *International Tax and Public Finance* 6(4): 459–472.

Lane, P.R, and G. Milesi-Ferretti. 2002. "External Wealth, the Trade Balance, and the Real Exchange Rate." *European Economic Review* 46: 1049-1071.

Lane, P.R, and G. Milesi-Ferretti. 2005. "Financial Globalization and Exchange Rates" IMF Working Paper 05/3. Washington, DC, United States: International Monetary Fund.

Lane, P.R., and G.M. Milesi-Ferretti. 2007. "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970–2004." *Journal of International Economics* 73: 223-250. Available at: http://www.philiplane.org/EWN.html

Lee, J. et al. 2008. "Exchange Rate Assessments: CGER Methodologies." IMF Occasional Paper 261. Washington, DC, United States: International Monetary Fund.

Phelps, E. 1961. "The Golden Rule of Accumulation: A Fable for Growthmen." *American Economic Review* 51(4): 638-643.

Phelps, E. 1965. "Second Essay on the Golden Rule of Accumulation." *American Economic Review* 55(4): 793-814.

Ramsey, F.P. 1928. "A Mathematical Theory of Saving." *Economic Journal* 38(152): 543-559.

Ricci, L., G-M. Milesi-Ferretti and J. Lee. 2008. "Real Exchange Rates and Fundamentals: A Cross-Country Perspective." IMF Working Paper 08/13. Washington, DC, United States: International Monetary Fund.

Tokarick, S. 2010. "A Method for Calculating Export Supply and Import Demand Elasticities." IMF Working Paper 10/180. Washington, DC, United States: International Monetary Fund.

Appendix
Table A1. Description of Variables to be Entered in the Toolkit

Name of the Variables	Definition	Cells	Sources
U.S. Consumer Price Index	U.S. Consumer Price Index end of per. base year = 1	C9	WEO or Official Sources
Nominal GDP, in USD	Nominal Gross Domestic Product, in USD	D9	
Real GDP Growth Rate in LCU (%)	Real GDP Annual Change, in LCU (in percent)	E9	
Real Effective Exchange Rate (INS/IMF) (+) Overvaluation, (-) undervaluation)	Real Effective Exchange Rate	F9	INS/IFS
Domestic Consumer Price Index	Domestic Consumer Price Index end of per. base year = 1	G9	WEO or Official Sources
Net Foreign Assets	Net Foreign Assets (Current US$)	H9	International Investment Position (IIP) Data from IMF or Official Sources and updates of Lane and Milesi-Ferretti (2007).
External Assets	Foreign External Assets, (Current US$)	I9	
External Liabilities	Foreign External Liabilities, (Current US$)	J9	
Current Account Balance	Current Account Balance in USD	K9	Balance of Payments, WEO, IFS, or Official Sources
Net Current Transfers	Net Current Transfers in USD, Current Account Balance	L9	
Trade Balance (goods plus services)	Trade Balance (goods plus services) in USD, Current Account Balance	M9	
Income Balance	Income Balance in USD, Current Account Balance	N9	
Exports Goods and Services[18]	Exports Goods and Services in USD, Current Account Balance	O9	
Imports Goods and Services	Imports Goods and Services in USD, Current Account Balance	P9	
Net Capital Transfers	Net Capital Transfers (Capital Account, e.g. Debt Forgiveness, Grants) in USD, Capital Account Balance	Q9	BOP/IFS or Official Sources
Portfolio Equity Assets Stock	Portfolio Equity Assets Stock, Foreign Assets	V9	International Investment Position (IIP) Data from IMF, Official Sources and updates of Lane and Milesi-Ferretti (2007).
FDI Assets Stock	Foreign Direct Investment Assets Stock, Foreign Assets	W9	
External Debt Assets Stock	External Debt (Portfolio Debt + Other Investment) Assets Stock, Foreign Assets	X9	
Financial Derivatives Assets Stock	Financial Derivatives Stock, Foreign Assets	Y9	
Reserves minus gold	Reserves minus gold, Foreign Assets	Z9	
Portfolio Equity Liabilities Stock	Portfolio Equity Liabilities Stock, Foreign Liabilities	AA9	
FDI Liabilities Stock	Foreign Direct Investment Liabilities Stock, Foreign Liabilities	AB9	
External Debt Liabilities Stock	External Debt (Portfolio Debt + Other Investment) Liabilities Stock, Foreign Liabilities	AC9	
Financial Derivatives Liabilities Stock	Financial Derivatives Liabilities Stock, Foreign Liabilities	AD9	
Investment Income Receipts (Income Credit)	Investment Income Receipts (Income Credit). International Investment Position (IMF) Data	AE9	International Financial Statistics (IFS-IMF) or Official Sources
Investment Income Payments (Income Debit)	Investment Income Payments (Income Debit). International Investment Position (IMF) Data	AF9	
Capital Outflows	International Capital Flows on Assets, Financial Account Balance	AG9	Balance of Payment (BOP) Data from IMF, Official Sources and updates of Lane and Milesi-Ferretti (2007).
Capital Inflows	International Capital Flows on Liabilities, Financial Account Balance	AH9	
US Treasury Bond Long-Run Real Yield	US Treasury Bond Long-Run Real Annualized Yield	AX9	Bloomberg. For other alternatives, see Box 2 in Castillo (2016.)
US Treasury Bond Long-Run Real Yield plus EMBI Yield	US Treasury Bond Long-Run Real Yield plus EMBI Yield	AY9	
BOP Real Rate of Return on Assets	IIP and BOP derived Real Rate of Return on Assets (Real Capital Gains inclusive)	AZ9	Calculation and Sources explained in detail in Section 3 of Castillo (2016).
BOP Real Rate of Return on Liabilities	IIP and BOP derived Real Rate of Return on Liabilities (Real Capital Gains inclusive)	BA9	

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[18] Exports and Imports Goods and Services are required to calculate the Elasticity of Trade Balance over GDP to REER explained in Section 2.2.2.